HEAT TRANSFER PROCESSES IN FLAT PLATE COLLECTORS

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JULY, 1977

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A Thesis Submitted
In Partial Fulfilment of the Requirements
for the degree of
MASTER OF TECHNOLOGY

By
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to the
DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
JULY, 1977

CERTIFICATE

Certified that the present work 'Heat transfer processes in flat plate collectors' by Kiran Kumar Sahu has been carried out under my supervision and has not been submitted elsewhere for the award of a degree.

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ACKNOWLEDGEMENTS

I am deeply indebted to Dr. J. Srinivasan for his able guidance at all stages of this work.

I thank all the persons, Mr. N.K. Jain in particular, who have helped me, directly or indirectly, in completing the present work.

I also thank Mr. G.L. Misra and Mr. S.K. Tewari for the trouble they have taken in typing this thesis which proved to be a rather tedious affair.

Abstract

The heat transfer processes in a flat plate collector have been analysed considering the spectral variation of the radiative properties of glass, fluid and collector plate and also the incoming solar radiation. The assumption of isothermal glass plate has been justified by a detailed analysis. A numerical method has been developed to calculate the variation of temperature of the glass, fluid and collector along the length. The effect of selective surface, glass thickness, glass-collector spacing, fluid (air and water) and Reynolds number on the efficiency is discussed. The fact that the flow is not fully developed is shown to be extremely important. The advantage in using non-dimensional numbers is also brought out.

TABLE OF CONTENTS

NOTATION

Symbol		Units
A	Constant as defined by Eqn. (2.24)	(non-dimensional)
B	Radiosity	(Watts/m^2)
Bo	Modified Biot number	(Non-dimensional)
C	Constant as defined by Eqn. (2.25)	(Non-dimensional)
D_H	Hydraulic diameter	(m)
Gz	Graetz number based on the hydraulic diameter	(Non-dimensional)
H	Irradiation	(Watts/m^2)
K	Thermal Conductivity	(Watts/m °K)
L	Length of flat plate collector	(m)
P	Function defined by Eqn. (2.4)	
Pr	Prandtl Number	(Non-dimensional)
Re	Reynolds number based on the hydraulic diameter	(Non-dimensional)
T	Absolute temperature	(°K)
V	Average velocity	
a	absorption coefficient	(m^{-1})
b	glass plate thickness	(m)
d	space between the glass plate and the collector plate	(m)
e	emissive power	(Watts/m^2)
h	heat transfer coefficient	(Watts/m^2 °K)
i	intensity of radiation from sources external to the glass plate	(Watts/m^2)

Symbol		Units
k	index of absorption	(m^{-1})
n	refractive index	(Non-dimensional)
p	as defined by Eqn. (2.4)	
q	heat flux	($Watts/m^2$)
r	reflection distribution function; effective reflectance	
t	transmission distribution function, effective transmittance	
x	geometrical co-ordinate	
y	geometrical co-ordinate	
z	geometrical co-ordinate.	

Greek Symbols

α	absorptivity, effective absorptance	(Non-dimensional)
β	function defined by Eqn. (A.2)	
γ	density	(Kg/m^3)
ϵ	emissivity, effective emittance	(Non-dimensional)
η	dummy variable	
ξ	dummy variable	
θ	polar angle	
κ	optical co-ordinate, κ^o-optical thickness	(Non-dimensional)
μ	polar direction cosine, $= \cos \theta$	(Non-dimensional)
ν	frequency of radiation	(cycles/sec.)
ρ	reflectivity	(Non-dimensional)
σ	Stefan-Boltzmann constant	($Watts/m^2\ {}^oK^4$)

Symbol		Units
τ	transmittivity	(Non-dimensional)
ϕ	azimuth angle	
Ω	solid angle	
λ	wavelength of radiation	(Microns)

Subscripts

b	blackbody
c	collimated
ν, λ	spectral
d	diffuse
R	radiative
C	conductive
1,2	interface of the glass plate
3	collector plate conditions
g	glass plate conditions
f	fluid

Superscripts

i	radiation internal to an interface
o	radiation incident upon an interface
+	forward directed
−	backward directed
^	non-dimensional.

LIST OF FIGURES

Figure

Figure

CHAPTER 1

Introduction

The utilization of unconventional energy sources is going to be very important for meeting the future energy demands of our country. One of the most important unconventional energy source is, of course, the solar energy. The energy from the sun can be directly converted to electricity through solar cells or it can be first converted to heat and later to work through heat engines. The most popular method of collecting solar energy as heat is through the use of flat plate collectors. The flat plate collector will continue to dominate the field of solar energy collection devices on account of the simplicity of its construction and low cost.

The common flat plate collector consists of a blackened metallic surface covered with a glass plate (to trap infrared radiation) with a transparent fluid (Air) flowing between them. Most of the work on flat plate collectors was done in the early sixties [see, for example, Spanides (1961)] and interest in this area has increased in the last few years. In recent years several modifications have been suggested to improve the efficiency of flat plate collectors. They include, for example, double glass covers, spectrally and directionally selective surfaces, radiatively absorbing 'black liquids', plastics etc. At present not much work has been done to quantify the improvement in the efficiency of the collector on account of these modifications. The primary reason for this is the insufficient understanding of the basic

heat transfer mechanisms. The analysis of the performance of a flat plate collector is complicated by the fact that the conduction, convection, and radiation heat transfer processes interact in a complex fashion (since they occur both in series and in parallel). Major improvements in flat plate collector design can be envisaged only after we understand the precise roles played by the three heat transfer modes in determining the heat loss and gain by the collector.

The first attempt to analyse the performance of flat plate collectors was made by Hottel and Woertz (1942) in which they considered the cover plates to be isothermal with the fluid flowing in tubes attached below the collector plate. Cobble (1964) determined the temperature field within a transparent solid irradiated by a Planckian, or a modified Planckian source. He proposed a simplified model for glass which absorbed monochromatic radiation according to Beer's law. However, he considered the other interface to be insulated. Safdari (1966) considered the transient heat transfer in a transparent material in contact with a well stirred fluid placed in a container. Lumsdaine (1970) analysed the heat transfer through a semi-transparent medium in contact with a fluid which absorbed all incident radiation. He also discussed the influence on fluid temperature of the reradiation from the solid, the relative importance of the ambient convective heat transfer coefficient and the effect of various physical parameters such as solid thickness, thermal conductivity and the absorption coefficient. He did not, however, consider the effect of the variation of temperature along the length of the collector.

In the present thesis a detailed analysis of heat transfer processes in a flat plate collector has been made by taking the following important aspects into consideration .-

1) Isothermal Glass Plate - In most of the previous work on flat plate collectors the assumption of the isothermal glass plate has been made without a proper justification. In Chapter 2 we shall justify this assumption by a rigorous analysis based on the work of Viskanta and Anderson (1975).

2) Spectral Nature of Glass :- The spectral nature of glass is considered as it will help us in evaluating the merits and demerits of various types of glass. A simplified model based on the work of Cobble (1964) is employed.

3) Spectral Nature of the Collector Plate - It is well known that the use of a spectrally selective surface can enhance the efficiency of a collector by reducing the infrared radiation emission. An analysis taking into account the spectral variation of the collector plate emissivity will show the effect of spectrally selective surfaces on the performance of flat plate collectors.

4) Developing Fluid Flow :- Most of the investigators in this area have assumed the flow of the fluid to be fully developed. However, for short flat plate collectors the flow will not be fully developed and, thus, we have to include the effect of non-fully developed region.

5) Non-dimensional Numbers :- The experimental data that is available on flat plate collectors is not easy to evaluate because most of them are presented in dimensional form. The advantages and insight

that can be gained by using non-dimensional numbers has not been fully utilized by workers in this field. As a matter of fact if the design of flat plate collectors has to be standardized, as has been done with conventional heat exchangers, then there is a need to identify the relevant non-dimensional parameters. In order to identify the relevant non-dimensional parameters we need to have a good understanding of the important heat gain and loss mechanisms.

CHAPTER 2

ANALYSIS FOR A FLAT PLATE COLLECTOR WITH TEMPERATURE VARYING ALONG THE THICKNESS

We shall restrict our attention to a flat plate collector (Fig. 1.1) in which a constant property, non-absorbing fluid flows in between the collector plateand the glass plate. The collector plate is a diffuse emitter of radiation. We assume that the back losses are zero. In practice, the back losses can be made to approach a zero value by providing a suitable insulation just below the collector plate. The top losses of the flat plate collector is dependent on the wind velocity and the conduction through the glass, of the energy emitted by the collector plate. Hence, the top losses cannot be made to approach a zero value and have to be considered in the analysis. The side losses are assumed to be negligible.

We shall determine the temperature distribution within the glass plate. To do so it is necessary to determine the temperatures of the fluid and the collector plate. These can be found from the energy balances for the fluid and collector plate respectively. We shall, therefore, write the equations of the energy balances for the fluid and the collector plate before determining the temperature distribution within the glass plate.

An attempt has also been made to determine the variation of the fluid, collector plate and glass plate temperatures along the length of the flat plate collector. This is done by dividing the flat plate collector into a number of small elements. The element nearest the entrance of the fluid is considered first. For the particular element

we determine the temperatures of the glass plate, the collector plate and the temperature of the fluid at the exit of the element. This fluid temperature is now used to determine the temperatures of the glass plate and the collector plate of the next element. We proceed in this manner until the exit of the fluid is reached.

In many collectors, since they are not very long, the flow is not fully developed and, therefore, one needs to account for the variation of the heat transfer coefficient. The variation of the convective heat transfer coefficients for a developing laminar or turbulent flow is taken from the analysis of Kays (1966). Moreover, the heat transfer coefficients at surface 2 of the glass plate and the collector plate have been assumed to be the same.

2.1) Energy Balance for the Fluid -

Consider a fluid element Δx at a distance x from the entrance. The total convective energy entering the control volume (Fig.1.2)

$$= \gamma_f \, C_{pf} \, d \, V_f \, T_f + h_3 \, (T_3 - T_f) \, \Delta x$$

The total convective energy leaving the control volume

$$= \gamma_f \, C_{pf} \, d \, V_f \, T_f + \frac{d}{dx} (\gamma_f \, C_{pf} \, d \, V_f \, T_f) \, \Delta x + h_2 \, (T_f - T_2) \, \Delta x$$

Applying the conservation of energy we get

$$\gamma_f \, C_{pf} \, d \, V_f \frac{dT_f}{dx} = h_3 \, (T_3 - T_f) - h_2 (T_f - T_2) \tag{2.1}$$

Eqn. (2.1) written in non-dimensional form is

$$\frac{d\hat{T}_f}{d\hat{x}} + 2\left[\frac{Nu_3(\hat{x}) + Nu_2(\hat{x})}{Gz}\right]\hat{T}_f = 2\left[\frac{Nu_3(\hat{x})\ \hat{T}_3(\hat{x}) + Nu_2(\hat{x})\ \hat{T}_2(\hat{x})}{Gz}\right] \qquad (2.2)$$

where $\hat{T}_f = T_f/T_\infty$, $\hat{x} = x/L$, $\Delta\hat{x} = \Delta x/L$, $Nu_3 = \frac{2h_3\ d}{K_f}$, $Nu_2 = \frac{2h_2\ d}{K_f}$, $Gz = 2Re\ Pr\ d/L$.

The integrating factor is $\exp\left[\int_0^{\hat{x}} g(\xi)\ d\xi\right]$ and $g(\xi) = 2\left[\frac{Nu_3(\xi)+Nu_2(\xi)}{Gz}\right]$

The solution to Eqn. (2.2) is

$$\hat{T}_f(\hat{x}) = \hat{T}_f(0)\ \exp\ [-p(\hat{x})] + \exp\ [-p(\hat{x})]\left[\int_0^{\hat{x}} P(\eta)\ d\eta\right] \qquad (2.3)$$

where $p(\hat{x}) = \int_0^{\hat{x}} g(\xi)\ d\xi$

$$\text{and } P(\eta) = 2\left[\frac{Nu_3(\eta)\ \hat{T}_3(\eta) + Nu_2(\eta)\ \hat{T}_2(\eta)}{Gz}\right]\exp\left[\int_0^{\eta} g(\xi)d\xi\right] \quad (2.4)$$

The temperature of the fluid at $\hat{x} + \Delta\hat{x}$ is given by

$$\hat{T}_f(\hat{x} + \Delta\hat{x}) = \hat{T}_f(0)\ \exp\ [-p(\hat{x} + \Delta\hat{x})] + \exp\ [-p(\hat{x} + \Delta\hat{x})]\left[\int_0^{\hat{x}+\Delta\hat{x}} P(\eta)d\eta\right] \qquad (2.5)$$

Subtracting Eqn. (2.3) from Eqn. (2.5) and expanding $p(\hat{x} + \Delta\hat{x})$ in a Taylor's series (neglecting second and higher order terms) we get

$$\begin{aligned}\hat{T}_f(\hat{x} + \Delta\hat{x}) - \hat{T}_f(\hat{x}) &= \hat{T}_f(0)\ \left[\exp\left\{-\frac{dp}{d\hat{x}}\Delta\hat{x}\right\} - 1\right]\exp\ [-p(\hat{x})] \\ &+ \exp\ [-p(\hat{x})]\ \exp\left(-\frac{dp}{d\hat{x}}\Delta\hat{x}\right)\left[\int_0^{\hat{x}} P(\eta)\ d\eta + \int_{\hat{x}}^{\hat{x}+\Delta\hat{x}} P(\eta)\ d\eta\right] \\ &- \exp\ [-p(\hat{x})]\int_0^{\hat{x}} P(\eta)\ d\eta \end{aligned} \qquad (2.6)$$

If $\frac{dp}{d\hat{x}} \Delta\hat{x} << 1$ we may replace $\exp(-\frac{dp}{d\hat{x}} \Delta\hat{x})$ by its series. Neglecting second and higher order terms Eqn. (2.6) reduces to

$$\hat{T}_f(\hat{x} + \Delta\hat{x}) = \hat{T}_f(\hat{x}) - \hat{T}_f(0) \exp\left[-p(\hat{x})\right] \frac{dp}{d\hat{x}} \Delta\hat{x}$$

$$+ \exp\left[-p(\hat{x})\right] \left\{1 - \frac{dp}{d\hat{x}} \Delta\hat{x}\right\} \left\{ \int_0^{\hat{x}} P(\eta)\, d\eta + P(\hat{x})\, \Delta\hat{x} \right\}$$

$$- \exp\left[-p(\hat{x})\right] \int_0^{\hat{x}} P(\eta)\, d\eta \tag{2.7}$$

or $$\hat{T}_f(\hat{x} + \Delta\hat{x}) = \hat{T}_f(\hat{x}) \left[1 - \frac{dp}{d\hat{x}} \Delta\hat{x}\right] + P(\hat{x})\, \Delta\hat{x} \exp\left[-p(\hat{x})\right] + O(\Delta x^2)$$

or $$\hat{T}_f(\hat{x} + \Delta\hat{x}) = \hat{T}_f(\hat{x}) \left[1 - 2\left\{\frac{Nu_3(\hat{x}) + Nu_2(\hat{x})}{Gz}\right\} \Delta\hat{x}\right]$$

$$+ 2\left[\frac{Nu_3(\hat{x})\, \hat{T}_3(\hat{x}) + Nu_2(\hat{x})\, \hat{T}_2(\hat{x})}{Gz}\right] \Delta\hat{x} \tag{2.8}$$

The above equation can be used to determine (numerically) the fluid temperature at any point provided the fluid, collector and glass temperatures are known at points upstream. The collector and glass temperatures can be determined only if the appropriate energy balance equations are written for them. This is done next.

2.2) Energy Balance for the Collector Plate

The radiant heat flux leaving the collector plate, $B_{3\nu}$, in a small frequency interval $d\nu$ about the frequency ν is given by

$$B_{3\nu}\, d\nu = \varepsilon_{3\nu}\, e_{b\nu}(T_3)\, d\nu + \rho_{3\nu}\, H_{3\nu}\, d\nu \tag{2.9}$$

where $H_{3\nu}$ is the radiant heat flux arriving at the collector plate.

Similarly, for the radiant heat flux leaving interface 2 of the glass plate, $B_{2\nu}$, we have

$$B_{2\nu}\, d\nu = \varepsilon_{g\nu}\, e_{b\nu}(T_2)\, d\nu + \rho_{g\nu}\, H_{2\nu}\, d\nu + q_{R\nu}(b)\, d\nu \qquad (2.10)$$

where $H_{2\nu}$ is the radiant heat flux arriving at interface 2 of the glass plate and $q_{R\nu}(b)$ is the radiant flux at interface 2 of the glass plate (see Appendix A).

We assume that all the radiant energy leaving a particular element of the collector plate reaches the corresponding element of interface 2 of the glass plate and vice-versa. Then

$$H_{3\nu}\, d\nu = B_{2\nu}\, d\nu \quad \text{and} \quad H_{2\nu}\, d\nu = B_{3\nu}\, d\nu \qquad (2.11)$$

From Eqns. (2.9), (2.10) and (2.11) we get the following expressions for $B_{2\nu}$ and $B_{3\nu}$

$$B_{2\nu} = \frac{\varepsilon_{g\nu}\, e_{b\nu}(T_2) + \rho_{g\nu}\, \varepsilon_{3\nu}\, e_{b\nu}(T_3) + q_{R\nu}(b)}{1 - \rho_{g\nu}\, \rho_{3\nu}} \qquad (2.12)$$

$$B_{3\nu} = \frac{\varepsilon_{3\nu}\, e_{b\nu}(T_3) + \rho_{3\nu}\, \varepsilon_{g\nu}\, e_{b\nu}(T_2) + \rho_{3\nu}\, q_{R\nu}(b)}{1 - \rho_{g\nu}\, \rho_{3\nu}} \qquad (2.13)$$

As the collector plate is insulated, the total gain in radiant energy by the collector plate must be equal to the total energy lost by convection.

$$\therefore\; h_3\, (T_3 - T_f) = \int_0^{\infty} (H_{3\nu} - B_{3\nu})\, d\nu \qquad (2.14)$$

$$\text{or } h_3(T_3-T_f) = \int_0^\infty \frac{(1-\rho_{3\nu})\ \varepsilon_{g\nu}\ e_{b\nu}(T_2)}{1-\rho_{g\nu}\rho_{3\nu}}\,d\nu - \int_0^\infty \frac{\varepsilon_{3\nu}(1-\rho_{g\nu})e_{b\nu}(T_3)}{1-\rho_{g\nu}\ \rho_{3\nu}}\,d\nu$$

$$+ \int_0^\infty \frac{(1-\rho_{3\nu})\ q_{R\nu}(b)}{1-\rho_{g\nu}\ \rho_{3\nu}}\,d\nu$$

The integration over frequency can be resolved into two ranges i.e., the opaque and the transparent. This facilitates the dropping of some of the terms. For most glasses the transparent region lies between 0.4 and 4μ while the opaque region exists above 4μ.

$$\text{or } h_3(T_3-T_f) = \int_{\Delta\nu_{\text{trans.}}} \frac{(1-\rho_{3\nu})\ \varepsilon_{g\nu}\, e_{b\nu}(T_2)}{1-\rho_{g\nu}\ \rho_{3\nu}}\,d\nu$$

$$- \int_{\Delta\nu_{\text{trans.}}} \frac{\varepsilon_{3\nu}(1-\rho_{g\nu})\ e_{b\nu}(T_3)}{1-\rho_{g\nu}\rho_{3\nu}}\,d\nu + \int_{\Delta\nu_{\text{trans.}}} \frac{(1-\rho_{3\nu})q_{R\nu}}{1-\rho_{g\nu}\ \rho_{3\nu}}\,d\nu$$

$$+ \int_{\Delta\nu_{\text{op.}}} \frac{(1-\rho_{3\nu})\ \varepsilon_{g\nu}\ e_{b\nu}(T_2)}{1-\rho_{g\nu}\rho_{3\nu}}\,d\nu - \int_{\Delta\nu_{\text{op.}}} \frac{\varepsilon_{3\nu}(1-\rho_{g\nu})e_{b\nu}(T_3)}{1-\rho_{g\nu}\rho_{3\nu}}\,d\nu$$

$$+ \int_{\Delta\nu_{\text{op.}}} \frac{(1-\rho_{3\nu})\ q_{R\nu}(b)}{1-\rho_{g\nu}\ \rho_{3\nu}}\,d\nu \qquad (2.15)$$

The last term in Eqn. (2.15) is equal to zero as the transmittivity of glass in the opaque regions is zero. As the fractions of the radiant energy lying in the transparent regions (0.4μ to 4μ for most glasses) for the glass and the collector plates are very small, the first and second terms are negligible compared to the other terms.

$$\therefore\ h_3(T_3-T_f) = \int_{\Delta\nu_{trans.}} \frac{(1-\rho_{3\nu})\ q_{R\nu}(b)}{1-\rho_{g\nu}\rho_{3\nu}}\, d\nu$$

$$+ \int_{\Delta\nu_{op.}} \frac{\varepsilon_{g\nu}\,\varepsilon_{3\nu}\ [e_{b\nu}(T_2) - e_{b\nu}(T_3)]}{\varepsilon_{g\nu} + \varepsilon_{3\nu} - \varepsilon_{g\nu}\,\varepsilon_{3\nu}}\, d\nu \qquad (2.16)$$

From Eqn. (2.16) we do not get the value of the collector temperature, T_3, explicitly. In order to do so we make use of the concept of the radiative heat transfer coefficient and replace the last term in Eqn. (2.16) by $h_{3r}\ (T_2 - T_3)$ where

$$h_{3r} = \left[\frac{\varepsilon_{g\nu}\,\varepsilon_{3\nu}}{\varepsilon_{g\nu} + \varepsilon_{3\nu} - \varepsilon_{g\nu}\,\varepsilon_{3\nu}}\right]_{op.} \sigma\,(T_2^2 + T_3^2)\ (T_2 + T_3) \qquad (2.17)$$

$\therefore$ Eqn. (2.16) reduces to

$$h_3\ (T_3 - T_f) = \int_{\Delta\nu_{trans.}} \frac{(1-\rho_{3\nu})\ q_{R\nu}(b)}{1-\rho_{g\nu}\,\rho_{3\nu}}\, d\nu + h_{3r}\ (T_2 - T_3) \qquad (2.18)$$

$$\text{or}\quad h_3\ (T_3 - T_2) = \int_{\Delta\nu_{trans.}} \frac{(1-\rho_{3\nu})\ q_{R\nu}(b)}{1-\rho_{g\nu}\,\rho_{3\nu}}\, d\nu + h_{3r}(T_2-T_3) + h_3(T_f-T_2)$$

$$\text{or}\quad (T_3-T_2) = \frac{1}{h_{3r}+h_3}\left[h_3(T_f-T_2) + \int_{\Delta\nu_{trans.}} \frac{(1-\rho_{3\nu})q_{R\nu}(b)}{1-\rho_{g\nu}\,\rho_{3\nu}}\, d\nu\right] \qquad (2.19)$$

2.3) <u>Temperature Distribution within the Glass Plate</u>

The general energy equation for steady state and in the absence of internal heat generation simplifies to

$$\frac{d}{dy}\ (q_C + q_R) = 0 \qquad (2.20)$$

where $q_C = -K_g \frac{dT}{dy}$ and $q_R = \int_0^{\infty} q_{R\nu}\, d\nu$ (2.21)

In order to reduce Eqn. (2.20) into a non-dimensional form, let

$$\hat{y} = y/b, \quad \hat{T} = T/T_\infty, \quad \hat{q}_R = q_R/q_S \tag{2.22}$$

where q_S denotes the total direct solar heat flux.

$$\therefore \quad \frac{d^2\hat{T}}{d\hat{y}^2} = \frac{q_S\, b}{K_g T_\infty} \frac{d\,\hat{q}_R}{d\hat{y}} \tag{2.23}$$

Integrating twice with respect to $\hat{y}$ we get

$$\frac{d\hat{T}}{d\hat{y}} = \frac{q_S\, b}{K_g T_\infty} \hat{q}_R + A \tag{2.24}$$

and

$$\hat{T} = \frac{q_S\, b}{K_g T_\infty} \int \hat{q}_R\, d\hat{y} + A\hat{y} + C \tag{2.25}$$

Eqn. (2.25) specifies the non-dimensional temperature distribution within the glass plate. A and C are constants which are to be determined from the boundary conditions at the two interfaces of the glass plate. The boundary conditions are

$$K_g \left.\frac{dT}{dy}\right|_{y=0} = h_1(T_1 - T_\infty) - \int_{\Delta\nu_{op.}} \varepsilon_{g\nu}\, i^o_{1c\nu}\, d\nu - \int_{\Delta\nu_{op.}} \pi\, \varepsilon_{g\nu}\, i^o_{1d\nu}\, d\nu$$

$$+ \int_{\Delta\nu_{op.}} \varepsilon_{g\nu} \left[e_{b\nu}(T_1) - e_{b\nu}(T_\infty) \right] d\nu \tag{2.26}$$

and

$$K_g \left.\frac{dT}{dy}\right|_{y=b} = h_2(T_f - T_2) + \int_{\Delta\nu_{op.}} \frac{\varepsilon_{g\nu}\, \varepsilon_{3\nu} \left[e_{b\nu}(T_3) - e_{b\nu}(T_2) \right]}{\varepsilon_{g\nu} + \varepsilon_{3\nu} - \varepsilon_{g\nu}\, \varepsilon_{3\nu}}\, d\nu \tag{2.27}$$

Eqn. (2.27) contains the fluid temperature, T_f, the collector plate temperature, T_3, and the Planck's function of the collector temperature. They can be found from the equations of the energy balance for the fluid and the collector plate respectively.

Using the radiative heat transfer coefficient concept we may rewrite Eqns. (2.26) and (2.27) as

$$K_g \left.\frac{dT}{dy}\right|_{y=0} = (h_1 + h_{1r})(T_1 - T_\infty) - \int_{\Delta\nu_{op.}} \varepsilon_{g\nu}\, i^o_{1c\nu}\, d\nu - \int_{\Delta\nu_{op.}} \pi\, \varepsilon_{g\nu}\, i^o_{1d\nu}\, d\nu \tag{2.28}$$

$$\text{and} \quad K_g \left.\frac{dT}{dy}\right|_{y=b} = h_2(T_f - T_2) + h_{3r}\,(T_3 - T_2) \tag{2.29}$$

$$\text{where} \qquad h_{1r} = [\varepsilon_{g\nu}]_{op.}\ \sigma(T_1^2 + T_\infty^2)\ (T_1 + T_\infty) \tag{2.30}$$

Substitution of Eqn. (2.19) in Eqn. (2.29) results in

$$K_g \left.\frac{dT}{dy}\right|_{y=b} = \left[h_2 + \frac{h_3\, h_{3r}}{h_{3r} + h_3}\right] (T_f - T_2) + \frac{h_{3r}}{h_3 + h_{3r}} \int_{\Delta\nu_{trans.}} \frac{(1-\rho_{3\nu})\, q_{R\nu}(b)}{1-\rho_{g\nu}\,\rho_{3\nu}}\, d\nu \tag{2.31}$$

Eqns. (2.28) and (2.31) in non-dimensional form are

$$\left.\frac{d\hat{T}}{d\hat{y}}\right|_{\hat{y}=0} = Bo_1\,(\hat{T}_1 - 1) - \frac{q_S\, b}{K_g T_\infty} \left[\int_{\Delta\nu_{op.}} \frac{\varepsilon_{g\nu}\, i^o_{1c\nu}}{q_S} d\nu + \frac{q_d}{q_S} \int_{\Delta\nu_{op.}} \pi\, \frac{\varepsilon_{g\nu}\, i^o_{1d\nu}}{q_d}\, d\nu\right] \tag{2.32}$$

$$\text{and} \quad \left.\frac{d\hat{T}}{d\hat{y}}\right|_{\hat{y}=1} = Bo_2\,(\hat{T}_f - \hat{T}_2) + \frac{q_S\, b}{K_g T_\infty}\ \frac{h_{3r}}{h_3 + h_{3r}} \int_{\Delta\nu_{trans.}} \frac{(1-\rho_{3\nu})\,\hat{q}_{R\nu}(1)}{1 - \rho_{g\nu}\,\rho_{3\nu}}\, d\nu \tag{2.33}$$

where the modified Biot numbers are defined as

$$Bo_1 = \left(\frac{h_1 + h_{1r}}{K_g}\right) b \quad \text{and} \quad Bo_2 = \left(h_2 + \frac{h_3\, h_{3r}}{h_3 + h_{3r}}\right) \frac{b}{K_g} \tag{2.34}$$

On substitution of Eqn. (2.24) into Eqns. (2.32) and (2.33)

$$\frac{q_S\, b}{K_g T_\infty} \hat{q}_R\Big|_{\hat{y}=0} + A = Bo_1(\hat{T}_1 - 1) - \frac{q_S b}{K_g T_\infty}\left[\int_{\Delta\nu_{op.}} \frac{\varepsilon_{g\nu}\, i^o_{1c\nu}}{q_S}\, d\nu + \frac{q_d}{q_S}\int_{\Delta\nu_{op.}} \pi\frac{\varepsilon_{g\nu}\, i^o_{1d\nu}}{q_d}\, d\nu\right] \tag{2.35}$$

and
$$\frac{q_S\, b}{K_g T_\infty} \hat{q}_R\Big|_{\hat{y}=1} + A = Bo_2(\hat{T}_f - \hat{T}_2) + \frac{q_S\, b}{K_g T_\infty}\left(\frac{h_{3r}}{h_3 + h_{3r}}\right)\int_{\Delta\nu_{trans.}} \frac{(1-\rho_{3\nu})\hat{q}_{R\nu}(1)}{1 - \rho_{g\nu}\rho_{3\nu}}\, d\nu \tag{2.36}$$

where q_d is the total diffuse solar radiation flux.

From Eqn. (2.25) we may say that

$$\hat{T}_1 = \hat{T}\big|_{\hat{y}=0} = \frac{q_S\, b}{K_g T_\infty}\int \hat{q}_R\, d\hat{y}\Big|_{\hat{y}=0} + C \tag{2.37}$$

and
$$\hat{T}_2 = \hat{T}\big|_{\hat{y}=1} = \frac{q_S\, b}{K_g T_\infty}\int \hat{q}_R\, d\hat{y}\Big|_{\hat{y}=1} + A + C \tag{2.38}$$

On substitution of Eqn. (2.37) in Eqn. (2.35) and Eqn. (2.38) in Eqn. (2.36) we get the following two equations in terms of the constants A and C

$$\frac{q_S\, b}{K_g T_\infty}\,\hat{q}_R\Big|_{\hat{y}=0} + A = Bo_1\,\{\frac{q_S\, b}{K_g T_\infty}\int \hat{q}_R\, d\hat{y}\Big|_{\hat{y}=0} + C - 1\}$$

$$- \frac{q_S\, b}{K_g T_\infty}\left[\int_{\Delta\nu_{op.}} \frac{\varepsilon_{g\nu}\, I^o_{1c\nu}}{q_S}\, d\nu + \frac{q_d}{q_S}\int_{\Delta\nu_{op.}} \pi\, \frac{\varepsilon_{g\nu}\, I^o_{1d\nu}}{q_d}\, d\nu\right] \tag{2.39}$$

$$\frac{q_S\, b}{K_g T_\infty}\,\hat{q}_R\Big|_{\hat{y}=1} + A = Bo_2\,\{\hat{T}_f - \frac{q_S\, b}{K_g T_\infty}\int \hat{q}_R d\hat{y}\Big|_{\hat{y}=1} - A - C\}$$

$$+ \frac{q_S\, b}{K_g T_\infty}\,\{\frac{h_{3r}}{h_3 + h_{3r}}\}\int_{\Delta\nu_{trans.}} \frac{(1-\rho_{3\nu})\hat{q}_{R\nu}(1)}{1 - \rho_{g\nu}\,\rho_{3\nu}}\, d\nu \tag{2.40}$$

From the above two equations we get

$$A = \Big[\, Bo_1 Bo_2(\hat{T}_f - 1) + \frac{q_S\, b}{K_g T_\infty}\,\{ Bo_2(Bo_1 \int \hat{q}_R\, d\hat{y}\Big|_{\hat{y}=0} - \hat{q}_R\Big|_{\hat{y}=0}$$

$$- \int_{\Delta\nu_{op.}} \frac{\varepsilon_{g\nu}\, I^o_{1c\nu}}{q_S}\, d\nu - \frac{q_d}{q_S}\int \pi\, \frac{\varepsilon_{g\nu}\, I^o_{1d\nu}}{q_d}\, d\nu) - Bo_1(\hat{q}_R\Big|_{\hat{y}=1}$$

$$+ Bo_2\int \hat{q}_R\, d\hat{y}\Big|_{\hat{y}=1} - \frac{h_{3r}}{h_3 + h_{3r}}\int_{\Delta\nu_{trans.}} \frac{(1-\rho_{3\nu})\hat{q}_{R\nu}(1)}{1 - \rho_{g\nu}\,\rho_{3\nu}}\, d\nu)\}\Big] \Big/$$

$$(Bo_1 + Bo_2 + Bo_1\, Bo_2) \tag{2.41}$$

and

$$C = 1 - \left[\frac{q_S\, b}{K_g T_\infty} \left\{ (1+Bo_2)(Bo_1 \int \hat{q}_R\, d\hat{y}|_{\hat{y}=0} - \hat{q}_R|_{\hat{y}=0} - \int_{\Delta\nu_{op.}} \frac{\varepsilon_{g\nu}\, i^o_{1c\nu}}{q_S}\, d\nu \right.\right.$$

$$- \frac{q_d}{q_S} \int_{\Delta\nu_{op.}} \pi \frac{\varepsilon_{g\nu}\, i^o_{1d\nu}}{q_d}\, d\nu) + Bo_2 \int \hat{q}_R\, d\hat{y}|_{\hat{y}=1} - Bo_2 (\hat{T}_f - 1)$$

$$\left.\left. + \hat{q}_R|_{\hat{y}=1} - \frac{h_{3r}}{h_3 + h_{3r}} \int_{\Delta\nu_{trans.}} \frac{(1 - \rho_{3\nu})\, \hat{q}_{R\nu}(1)}{1 - \rho_{g\nu}\, \rho_{3\nu}}\, d\nu \right\} \right] \Big/ (Bo_1 + Bo_2 + Bo_1 Bo_2) \qquad (2.42)$$

On substituting for the constants from Eqn. (2.41) and (2.42) respectively into Eqn. (2.25) we obtain the temperature distribution within the glass plate.

From the temperature profile within the glass plate shown in Fig. (2.1), we find that the temperature difference across the glass plate is not very large. This is because 1) the conductive resistance of the glass plate is small compared to the convective resistance at the outer surface 2) the contribution of solar absorption to the energy balance of the glass plate is not substantial and 3) the radiative energy exchange between glass and collector are almost equal and hence the <u>net</u> radiative transfer from collector to glass is negligible.

CHAPTER 3

ANALYSIS FOR A FLAT PLATE COLLECTOR WITH TEMPERATURE CONSTANT ALONG THE THICKNESS

The results of the previous chapter show that the temperature difference across the glass plate is not very large. We may, therefore, assume the glass plate to be isothermal. In this chapter we present an analysis of the flat plate collector performance assuming an isothermal glass plate. The other assumptions made are the same as that in Chapter 2. One of the major assumptions made in Chapter 2 was that the fluid flowing between the glass cover and the collector was transparent to radiation. This assumption is satisfied by air but not by water. A one centimeter thick layer of water is opaque to all radiation above one micron wavelength while being transparent to radiation below one micron wavelength.

3.1 Energy Balance for the Glass Plate:

The total radiant energy arriving at surface 1 of the glass plate, H_1, is the sum of the total direct and diffuse solar radiant energy and the radiant energy from the surrounding environment.

$$\therefore \quad H_1 = \int_0^\infty H_{1\nu}\, d\nu = \int_0^\infty i^o_{1c\nu}\, d\nu + \pi \int_0^\infty i^o_{1d\nu}\, d\nu + \int_0^\infty e_{b\nu}(T_\infty)\, d\nu \qquad (3.1)$$

The total radiant energy leaving surface 1 of the glass plate, B_1, is composed of the energy emitted by the glass plate, the fraction of the total radiant energy arriving at surface 1 that is reflected, and the fraction of the total radiant energy leaving the collector plate that is transmitted through the glass plate.

$$B_1 = \int_0^\infty B_{1\nu}\, d\nu = \int_0^\infty \varepsilon_{g\nu}\, e_{b\nu}(T_g)\, d\nu + \int_0^\infty r_{gc\nu}(\mu')\, i^o_{1c\nu}\, d\nu$$

$$+ \pi \int_0^\infty r_{gd\nu}\, i^o_{1d\nu}\, d\nu + \int_0^\infty e_{b\nu}(T_g)\, r_{gd\nu}\, d\nu + \int_0^\infty t_{gd\nu}\, B_{3\nu}\, d\nu \quad (3.2)$$

where $B_{3\nu}$ is the radiant energy leaving the collector plate in a small frequency interval $d\nu$ around the frequency ν. The equation for $B_{3\nu}$ is

$$B_{3\nu} = \varepsilon_{3\nu}\, e_{b\nu}(T_3) + (1 - \varepsilon_{3\nu})\, H_{3\nu} \quad (3.3)$$

The first term is the emission from the collector plate and the second term represents the fraction of the radiant energy arriving at the collector plate that is reflected. $H_{3\nu}$ is the energy arriving at the collector plate. In order to determine $H_{3\nu}$ we must first determine $B_{2\nu}$, the energy leaving surface 2 of the glass plate.

$$B_{2\nu} = \varepsilon_{g\nu}\, e_{b\nu}(T_g) + r_{gd\nu}\, H_{2\nu} + t_{gc\nu}(\mu')\, i^o_{1c\nu} + \pi\, t_{gd\nu}\, i^o_{1d\nu} + t_{gd\nu}\, e_{b\nu}(T_\infty) \quad (3.4)$$

The first term on the r.h.s. of Eqn. (3.4) represents the emission from surface 2, the second term represents the energy reflected by surface 2, and the rest represent the energy transmitted through the glass plate. $H_{2\nu}$ denotes the energy from the collector plate that arrives at surface 2 of the glass plate. If we make the same assumptions regarding the energy arriving at the collector plate and surface 2 of the glass plate as in Chapter 2 we have

$$H_{2\nu} = B_{3\nu} \text{ and } H_{3\nu} = B_{2\nu} \quad (3.5)$$

Substitution of Eqn. (3.5) into Eqns. (3.3) and (3.4) results in two equations in $B_{2\nu}$ and $B_{3\nu}$. Solving these we get the following expressions for $B_{2\nu}$ and $B_{3\nu}$

$$B_{2\nu} = [\, t_{gc\nu}(\mu')\, i^{o}_{1c\nu} + \pi\, t_{gd\nu}\, i^{o}_{1d\nu} + t_{gd\nu}\, e_{b\nu}(T_\infty) + \varepsilon_{g\nu}\, e_{b\nu}(T_g) + r_{gd\nu}\, \varepsilon_{3\nu}\, e_{b\nu}(T_3)\,] \,/\{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})\} \tag{3.6}$$

and

$$B_{3\nu} = [\varepsilon_{3\nu}\, e_{b\nu}(T_3) + (1 - \varepsilon_{3\nu})\{\varepsilon_{g\nu} e_{b\nu}(T_g) + t_{gc\nu}(\mu')\, i^{o}_{1c\nu} + \pi\, t_{gd\nu}\, i^{o}_{1d\nu} + t_{gd\nu}\, e_{b\nu}(T_\infty)\}] \,/\, \{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})\} \tag{3.7}$$

The gain in total radiant energy by the glass plate must be equal to the energy removed by convection.

$$h_2 (T_g - T_f) + h_1(T_g - T_\infty) = (H_1 - B_1) + (H_2 - B_2) \tag{3.8}$$

$$\text{where } H_2 = \int_0^\infty H_{2\nu}\, d\nu \quad \text{and} \quad B_2 = \int_0^\infty B_{2\nu}\, d\nu \tag{3.9}$$

On substitution of Eqns. (3.1), (3.2), (3.5), (3.6) and (3.7) into Eqn. (3.8) we get [see Eqn. (C.4) Appendix C]

$$h_2(T_g-T_f) + h_1(T_g-T_f) = \int_0^\infty \varepsilon_{g\nu}[e_{b\nu}(T_\infty) - e_{b\nu}(T_g)]\, d\nu$$

$$+ \int_0^\infty \frac{\varepsilon_{3\nu}\, \varepsilon_{g\nu}\,[e_{b\nu}(T_3) - e_{b\nu}(T_g)]}{1 - r_{gd\nu}\,(1 - \varepsilon_{3\nu})}\, d\nu$$

$$+ \int_0^\infty \frac{t_{gd\nu}\,(1 - \varepsilon_{3\nu})\,\varepsilon_{g\nu}[e_{b\nu}(T_\infty) - e_{b\nu}(T_g)]}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\, d\nu$$

$$+ \int_0^\infty \varepsilon_{g\nu}\,(\mu')\, i^o_{1c\nu}\, d\nu + \int_0^\infty \pi\, \varepsilon_{g\nu}\, i^o_{1d\nu}\, d\nu$$

$$+ \int_0^\infty \frac{t_{gc\nu}(\mu')\,(1 - \varepsilon_{3\nu})\,\varepsilon_{g\nu}\, i^o_{1c\nu}}{1 - r_{gd\nu}\,(1 - \varepsilon_{3\nu})}\, d\nu$$

$$+ \int_0^\infty \pi \frac{t_{gd\nu}\,(1 - \varepsilon_{3\nu})\,\varepsilon_{g\nu}\, i^o_{1d\nu}}{1 - r_{gd\nu}\,(1 - \varepsilon_{3\nu})}\, d\nu \qquad (3.10)$$

The first term on the right hand side of Eqn. (3.10) represents the direct radiant energy exchange between the surrounding environment and the glass plate, the second term that between

From Eqn. (3.12) we can determine the temperature of the isothermal glass plate provided we know the temperatures of the fluid and the collector plate. These temperatures are determined from the energy balances for the fluid and the collector plate respectively. The energy balance for the fluid is given by Eqn. (2.8). We, therefore, now proceed to the task of writing the energy balance for the collector plate.

3.2 Energy Balance for the Collector Plate

The gain in total radiant energy by the collector plate is, in the case of steady state, equal to the heat removed from it by convection.

$$h_3(T_3 - T_f) = H_3 - B_3 \tag{3.13}$$

Substitution of Eqns. (3.5), (3.6) and (3.7) into Eqn. (3.13) results in {see Eqn. (C.5) Appendix C}

$$\begin{aligned} h_3(T_3 - T_f) = &\int_0^\infty \frac{\varepsilon_{g\nu}\,\varepsilon_{3\nu}\,[e_{b\nu}(T_g) - e_{b\nu}(T_3)]}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\,d\nu \\ &+ \int_0^\infty \frac{\varepsilon_{3\nu}\,t_{gd\nu}\,[e_{b\nu}(T_\infty) - e_{b\nu}(T_3)]}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\,d\nu \\ &+ \int_0^\infty \frac{\varepsilon_{3\nu}\,t_{gc\nu}(\mu')\,i^o_{1c\nu}}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\,d\nu \\ &+ \pi\int_0^\infty \frac{\varepsilon_{3\nu}\,t_{gd\nu}\,i^o_{1d\nu}}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\,d\nu \end{aligned} \tag{3.14}$$

The first term on the right hand side of Eqn. (3.14) represents the radiant exchange of energy between the glass plate and the collector plate and the second that between the environment and the collector plate. The last two terms represent the solar energy absorbed by the collector plate.

Eqn. (3.14) may also be written as

$$h_3(T_3 - T_f) = \int_0^\infty \frac{\varepsilon_{g\nu}\, \varepsilon_{3\nu} \left[e_{b\nu}(T_g) - e_{b\nu}(T_3) \right]}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\, d\nu$$

$$+ \int_{\Delta\nu_{trans.}} \frac{\varepsilon_{3\nu}\, t_{gd\nu} \left[e_{b\nu}(T_\infty) - e_{b\nu}(T_3) \right]}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\, d\nu$$

$$+ \int_{\Delta\nu_{trans.}} \frac{\varepsilon_{3\nu}\, t_{gc\nu}(\mu')\, i^o_{1c\nu}}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\, d\nu$$

$$+ \pi \int_{\Delta\nu_{trans.}} \frac{\varepsilon_{3\nu}\, t_{gd\nu}\, i^o_{1d\nu}}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\, d\nu \qquad (3.15)$$

The third term and the second term on the right hand side of Eqns. (3.12) and (3.15) respectively are negligible as compared to the other terms. Moreover, the solution for the temperatures of the glass plate and the collector plate are not easily obtained from the above equations as they are non-linear in nature. We, therefore, linearize both equations (using the radiative heat transfer coefficient concept) as a first approximation. Eqns. (3.12) and (3.15) now reduce to

$$(h_1+h_2)\ T_g - (h_1\ T_\infty + h_2\ T_f) = h_{1r}(T_\infty - T_g) + h_{3r}(T_3 - T_g)$$

$$+ \int_0^\infty \varepsilon_{g\nu}\ (\mu')\ I^o_{1c\nu}\ d\nu + \pi \int_0^\infty \varepsilon_{g\nu}\ I^o_{1d\nu}\ d\nu$$

$$+ \int_{\Delta\nu_{trans.}} \frac{t_{gc\nu}(\mu')\ (1 - \varepsilon_{3\nu})\ \varepsilon_{g\nu}(\mu')\ I^o_{1c\nu}}{1 - r_{gd\nu}\ (1 - \varepsilon_{3\nu})}\ d\nu$$

$$+ \int_{\Delta\nu_{trans.}} \pi \frac{t_{gd\nu}\ (1 - \varepsilon_{3\nu})\ \varepsilon_{g\nu}\ I^o_{1d\nu}}{1 - r_{gd\nu}\ (1 - \varepsilon_{3\nu})}\ d\nu \qquad (3.16)$$

and

$$h_3(T_3-T_f) = h_{3r}\ (T_g-T_3) + \int_{\Delta\nu_{trans.}} \frac{\varepsilon_{3\nu}\ t_{gc\nu}(\mu')\ I^o_{1c\nu}}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\ d\nu$$

$$+ \int_{\Delta\nu_{trans.}} \pi \frac{\varepsilon_{3\nu}\ t_{gd\nu}\ I^o_{1d\nu}}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\ d\nu \qquad (3.17)$$

or $$T_g = \Big[\ (h_1+h_{1r})\ T_\infty + (h_2 + h_{3r})\ T_3 + \int_0^\infty \varepsilon_{g\nu}(\mu')\ I^o_{1c\nu}\ d\nu$$

$$+ \pi \int_0^\infty \varepsilon_{g\nu}\ I^o_{1d\nu}\ d\nu$$

$$+ \int_{\Delta\nu_{trans.}} \frac{t_{gc\nu}(\mu')\ (1 - \varepsilon_{3\nu})\ \varepsilon_{g\nu}(\mu')\ I^o_{1c\nu}}{1 - r_{gd\nu}\ (1 - \varepsilon_{3\nu})}\ d\nu$$

$$+ \int_{\Delta\nu_{trans.}} \pi \frac{t_{gd\nu}\ (1-\varepsilon_{3\nu})\ \varepsilon_{g\nu}\ I^o_{1d\nu}}{1 - r_{gd\nu}\ (1 - \varepsilon_{3\nu})}\ d\nu\Big] / (h_1+h_2+h_{1r}+h_{3r})$$

$$(3.18)$$

and

$$T_3 = \Big[h_3 T_f + h_{3r} T_g + \int_{\Delta\nu_{trans.}} \frac{\varepsilon_{3\nu}\, t_{gc\nu}(\mu')\, i^o_{1c\nu}}{1 - r_{gd\nu}\,(1 - \varepsilon_{3\nu})}\, d\nu$$

$$+ \int_{\Delta\nu_{trans.}} \pi \frac{\varepsilon_{3\nu}\, t_{gd\nu}\, i^o_{1d\nu}}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})}\, d\nu \Big] / (h_3 + h_{3r}) \qquad (3.19)$$

From Eqn. (3.19) we determine the collector temperature, T_3, in terms of the glass plate temperature, T_g. Substitution of this value of the collector temperature, T_3, in terms of the glass plate temperature, T_g, in Eqn. (3.18) results in an expression in the glass temperature and the fluid temperature, T_f. Using the value of the glass temperature, T_g, obtained from this expression we determine the collector temperature from Eqn. (3.15). This collector temperature is now used in Eqn. (3.12) to determine a new glass temperature. We now go back to Eqn. (3.15) and determine a new collector temperature. The process is repeated till a reasonable convergence of the glass plate and collector plate temperatures are obtained.

Once the temperature of the fluid at the exit is known, we can determine the efficiency of the flat plate collector from the equation given below

$$\text{Efficiency} = \frac{\gamma_f\, C_{pf}\, V_f\, d\, (T_{f,out} - T_{f,in})}{\left[\int_0^\infty i^o_{1c\nu}\, d\nu + \pi \int_0^\infty i^o_{1d\nu}\, d\nu \right] \times L} \qquad (3.20)$$

or on making the appropriate transformations

$$\text{Efficiency} = \frac{1}{2}\left(\frac{K_f}{D_H}\right) \times \left(\frac{Gz}{\int_0^\infty i^o_{1c\nu}\, d\nu + \pi \int_0^\infty i^o_{1d\nu}\, d\nu}\right) \times (T_{f,out} - T_{f,in}) \tag{3.21}$$

3.3 Radiatively Absorbing Fluid

The analysis given previously is not valid for an absorbing fluid like water. The absorption coefficient in the region less than 1μ is negligible as shown in Fig. (3.2) which is taken from Goody (1964). It is a reasonable assumption that water transmits all radiation at wavelengths below 1μ and it absorbs all radiation at wavelengths greater than 1μ. Hence, as far as the emission of radiation is concerned, water is assumed to behave like a blackbody. We also assume that the water is isothermal (at T_f) for the sake of simplicity of analysis. We now proceed to the task of writing the equations of the energy balances for the glass plate, the collector plate, and the fluid respectively based on this simplified model.

a) Energy balance for the glass plate :

The energy absorbed by the glass plate must be equal to the sum of the energy emitted by it and the convective energy removed from the two surfaces.

$$\therefore \quad \varepsilon_g\ (\sigma T_\infty^4 + \sigma T_f^4) = 2\ \varepsilon_g\ \sigma T_g^4 + h_1(T_g - T_\infty) + h_2(T_g - T_f) \tag{3.22}$$

b) Energy balance for the collector plate

The collector plate absorbs the solar energy transmitted first through the glass plate and then through water. It also absorbs energy emitted by the water. The loss is through emission and by convection at its surface. An energy balance, therefore, results in the following expression

$$\int_{0.4\mu}^{1.0\,\mu} \varepsilon_{3\nu}\, i^{o}_{1c\nu}\, d\nu + \pi \int_{0.4\mu}^{1.0\,\mu} \varepsilon_{3\nu}\, i^{o}_{1d\nu}\, d\nu + \varepsilon_3\, \sigma T_f^4 = \varepsilon_3\, \sigma T_3^4 + h_3(T_3 - T_f) \tag{3.23}$$

c) Energy balance for the fluid

An energy balance for a fluid element of length dx results in

$$\begin{aligned} &\gamma_f\, V_f\, d\, C_{pf}\, T_f + \{\varepsilon_g\, \sigma\, T_g^4 + \varepsilon_c\, \sigma T_c^4\}\, dx \\ &\quad + \{\int_{1\mu}^{4\mu} t_{gc\nu}\, i^{o}_{1c\nu}\, d\nu + \pi \int_{1\mu}^{4\mu} t_{gd\nu}\, i^{o}_{1d\nu}\, d\nu\}\, dx \\ &\quad = 2\, \sigma T_f^4\, dx + \gamma_f\, d\, V_f\, C_{pf}\, (T_f + \frac{d\, T_f}{dx})\, dx \\ &\quad + h_2(T_f - T_g) + h_3(T_f - T_3) \end{aligned} \tag{3.24}$$

$$\begin{aligned} \therefore\ \gamma_f\, V_f\, d\, C_{pf} \frac{dT_f}{dx} &= h_2\, (T_g - T_f) + h_3(T_3 - T_f) + \varepsilon_g\, \sigma\, (T_g^4 - T_f^4) \\ &\quad + \varepsilon_3\, \sigma (T_3^4 - T_f^4) + \int_{1\mu}^{4\mu} t_{gc\nu}\, i^{o}_{1c\nu}\, d\nu \\ &\quad + \int_{1\mu}^{4\mu} \pi\, t_{gd\nu}\, i^{o}_{1d\nu}\, d\nu \end{aligned} \tag{3.25}$$

Eqn. (3.25) is non-dimensional form is

$$\frac{d\hat{T}_f}{d\hat{x}} = \frac{2Nu_2(\hat{x})}{Gz}(\hat{T}_g - \hat{T}_f) + \frac{2Nu_3(\hat{x})}{Gz}(\hat{T}_3 - \hat{T}_f) + \frac{4\ \varepsilon_g\ \sigma T_\infty^3\ d}{Gz\ K_f}(\hat{T}_g^4 - \hat{T}_f^4)$$

$$+ \frac{4\ \varepsilon_3\ \sigma T_\infty^3\ d}{Gz\ K_f}\ \{\hat{T}_3^4 - \hat{T}_f^4\} + \frac{4q_S\ d}{K_f\ T_\infty\ Gz}\int_{1\mu}^{4\mu} t_{gc\nu}\frac{i^o_{1c\nu}}{q_S}d\nu$$

$$+ \frac{4g_d\ d}{K_f\ T_\infty\ Gz}\int_{1\mu}^{4\mu}\pi\, t_{gd\nu}\frac{i^o_{1d\nu}}{q_d}\ d\nu \tag{3.26}$$

We integrate Eqn. (3.26) to get $\hat{T}_f(\hat{x})$. Using the method given in Section (2.1) we get $\hat{T}_f(\hat{x} + \Delta\hat{x})$ as

$$\hat{T}_f(\hat{x} + \Delta\hat{x}) = \hat{T}_f(\hat{x}) + \frac{2Nu(\hat{x})}{Gz}\ \{\hat{T}_g(\hat{x}) + \hat{T}_3(\hat{x}) - 2\hat{T}_f(\hat{x})\}\ \Delta\hat{x}$$

$$+ \frac{4\sigma d}{Gz\ K_f}\left[\varepsilon_g\ \{(\hat{T}_g^4(\hat{x}) - \hat{T}_f^4(\hat{x})\} + \varepsilon_3\ \{(\hat{T}_3^4(\hat{x}) - \hat{T}_f^4(\hat{x})\}\right]$$

$$+ \frac{4q_S\ d}{K_f\ Gz}\int_{1\mu}^{4\mu} t_{gc\nu}\frac{i^o_{1c\nu}}{q_S}\ d\nu + \frac{4q_d\ d}{K_f\ T_\infty\ Gz}\int_{1\mu}^{4\mu}\pi\, t_{gd\nu}\frac{i^o_{1d\nu}}{q_d}d\nu \tag{3.27}$$

In Eqns. (3.22), (3.23) and (3.27) we have three non-linear expressions in the glass plate, collector plate and the fluid temperatures. These three unknown temperatures may be found using the procedure explained earlier.

CHAPTER 4

RESULTS AND DISCUSSIONS

The equations presented in the last chapter were solved numerically using the following input data :

1) The spectral variation of absorption coefficient and reflectivity for different types of glass were taken from Gleek (1959).
2) The heat transfer coefficient on the outer surface of glass was taken to be equal to $(5.7 + 3.8\ V_{amb.})$ W/m^2 °C as suggested by Duffie and Beckman (1974).
3) The variation of Nusselt number on the inner surface of glass and the collector was taken from the solution for developing flow given by Kays (1966). The expression for Nusselt number is given as

$$Nu(\hat{x}) = \frac{\sum G_n \exp\left(-\lambda_n^2 \frac{\hat{x}}{Gz}\right)}{2 \sum (G_n/\lambda_n^2) \exp(-\lambda_n^2\ \hat{x}/Gz)}$$ where λ_n and G_n

are eigenvalues and eigenfunctions respectively. These are tabulated in Kays (1966). In the case of laminar flow, where the fully developed Nusselt number is not a function of Reynolds number, the above expression can be used directly. For turbulent flow, however, we have to include the effect of Reynolds number on the fully developed Nusselt number. This was incorporated by multiplying the above expression by $(\frac{Re}{Re*})^{0.8}$ where Re* is the reference Reynolds number for which the eigenvalues and functions

were available.

4) The spectral variation of incoming direct solar radiation were taken from Coulson (1975) and that for diffuse radiation from Probert and Hub (1968).

From Fig. (4.1) and Fig. (4.2) we see that the efficiency of the collector increases continuously with the Reynolds number. This is because the outlet temperature (and hence the glass and collector temperatures) decrease as the Reynolds number increases and, hence, the convective and radiative losses to the ambient also decrease. At very large Reynolds numbers the efficiency asymptotically approaches a value around 75% for a water heater and around 80% for an air heater. In this limit the convective and radiative losses are small and the main losses are the optical losses due to reflection and absorption in glass.

Figs. (4.3) and (4.4) show the variation of the collector, glass and fluid temperatures along the lengtn of the collector for air and water respectively. We see that in the water heater, the collector temperature is greater than the fluid temperature which is greater than the glass temperature. In air heaters both the collector and glass temperatures are well above the fluid temperature. The high glass temperature implies higher heat losses to the ambient. Hence, air heaters are much less efficient than water heaters. In water heaters the temperature difference between the glass, collector and fluid is very small (around 2°C) while in air heaters the temperature

differences are of the order of 30°C. This is primarily due to the lower heat transfer coefficient of air as compared to water.

In Figs. (4.5) and (4.6) the increase of the temperature of the fluid (i.e., $T_{f,out} - T_{f,in}$) is plotted against the Reynolds number. We see that as the Reynolds number increases the outlet temperature decreases rapidly. As a matter of fact for a given application ($T_{f,out} - T_{f,in}$) will be specified and, hence, the Reynolds number is fixed. Further discussions on this point will be taken up later.

In Figs. (4.7) and (4.8) the effect of emissivity of the collector on the efficiency for both air and water heaters are shown. The efficiency decreases rapidly with the decrease in emissivity but interestingly the efficiency does not become zero as the emissivity of the collector goes to zero. This is because even when the collector reflects all the radiation, the water is able to absorb some of the solar radiation directly and in the case of an air heater some heat is transferred from glass (which absorbs solar radiation) to air.

In Fig. (4.9) the effect of the angle of incidence of direct solar radiation on the efficiency is shown. We see that when the angle is less than 45° ($\mu' > 0.7$) the efficiency does not decrease substantially. At angles greater than 45° the decrease in efficiency is very large and is on account of the rapid decrease in the transmittivity. A plot of the transmittivity (for direct radiation) with $\mu'(= \cos\theta')$ is shown in Fig. (4.18) and it is clearly seen that the trend shown here is the same as in Fig. (4.9).

In Fig. (4.10) the effect of ambient wind velocity on the efficiency of an air heater is shown. We see that as the ambient wind velocity increases the efficiency decreases rapidly. This is because the most dominant resistance to heat loss from the fluid is the convective resistance at the outer surface of the glass plate. As this resistance decreases (with increase in wind velocity) the heat losses increase rapidly.

In Fig. (4.11) we see the effect of glass thickness on efficiency. As the glass thickness increases the efficiency decreases rapidly on account of the decrease in transmittance of the solar radiation. One may be tempted to think that as the glass thickness increases the heat losses to the ambient should decrease on account of a greater conductive resistance. But as pointed out at the conclusion of Chapter 2 the conductive resistance of glass is negligible as compared to the convective resistance at the outer face of glass and, hence, an increase in glass thickness does not contribute to a decrease in heat losses but only to a decrease in transmittance of the solar radiation.

In Fig. (4.12) the spectral variation of absorption coefficient and reflectivity is shown. Glass A absorbs more of the solar radiation (in the region 0.4 to 2.7 μ) than Glass B. At higher Reynolds number the energy absorbed by the glass is transferred to the fluid more readily than at low Reynolds number. Hence at high Reynolds number the Glass A has a higher efficiency than Glass B.

In Figs. (4.13) and (4.14) we show the effect of the depth of flow on efficiency. As the ratio d/L increases, the efficiency decreases on account of decreasing heat transfer coefficient. In the case of air the flow is never fully-developed. In the case of water, the flow is fully developed for $\frac{d}{L}$ of 0.01. As $\frac{d}{L}$ is increased from 0.01 to 0.02, the heat transfer coefficient decreases and, hence, the efficiency decreases. Increasing $\frac{d}{L}$ from 0.02 to 0.03 does not result in a substantial change in efficiency because of two counteracting effects. As the ratio $\frac{d}{L}$ is increased the fully developed heat transfer coefficient decreases but at the same time the region of developing flow is more in the case of $\frac{d}{L} = 0.03$ than $\frac{d}{L} = 0.02$. It is well known that in the regions of developing flow the heat transfer coefficients are higher than the developed region. Hence, the above two effects cancel each other resulting in no substantial increase in efficiency.

Figs. (4.15), (4.16) and (4.17) show some characteristics of selective surfaces. The use of selective surfaces is usually resorted to in order to increase the efficiency of flat plate collectors. In Fig. (4.15) we compare the efficiency of the air heater for selective surfaces with various cut-off frequencies. We see that as the cut-off frequency is increased from 1 μ the efficiency increases. The increase in efficiency becomes smaller as we approach 4 μ. This is because most of the solar radiation lies in the region 0.4 μ - 4 μ and hence a selective surface which has a high absorptivity from 0 to 4 μ and low absorptivity above 4 μ (in which region the emission from the collector lies) is the

best choice. It is interesting to note that a collector with emissivity equal to one can have an efficiency greater or less than a collector with a selective surface. At very high Reynolds numbers the collector temperature is low and hence the radiative losses are also small. In this case a collector with emissivity equal to one is better than a selective surface because it absorbs all the solar radiation while the selective surface absorbs only 90% of it. In the low Reynolds number region, the collector temperature is large and, hence, the heat losses are substantial. Therefore, the selective surface (which has a low emissivity in the region greater than 4 μ) is superior to a black surface under these conditions. From the above discussion we see that at low Reynolds number (i.e., high outlet temperatures) we need a selective surface with a cut-off wavelength of 4 μ . All the selective surfaces developed so far have a cut-off wavelength around 1 μ (e.g. Silicon Oxide on Aluminium has a cut-off wavelength of about 1.6 μ). The above analysis thus shows that selective surfaces with higher cut-off wavelengths must be developed.

In Fig.(4.16) we show a comparison between a selective surface and a gray diffuse surface having an emissivity of 0.9. The gray diffuse surface can have an efficiency lower or higher than a selective surface having a cut-off wavelength of 1.0μ or 1.5μ depending on whether the Reynolds number is low or high. The reason for this behaviour has been explained above. However, a selective surface with a cut-off wavelength of 4 μ has a higher efficiency than that of a gray diffuse surface having an emissivity equal to 0.9. This is because, even though the solar radiation

absorbed by the two surfaces are equal, the radiative heat losses using a gray diffuse surface of emissivity 0.9 is greater than that when using a selective surface having a cut-off wavelength equal to 4μ.

In Fig. (4.17) a comparison between a black collector and a selective surface with a cut-off wavelength of 1μ is shown. We see that the black surface has a greater efficiency than a selective surface. Increasing the cut-off wavelength beyond 1μ will not bring about any change in the results as water completely absorbs all the radiant energy lying above 1μ.

In Fig. (4.19) we compare the present method with Hottel's method as presented in Duffie and Beckman (1974) . We see that Hottel's method predicts a lower efficiency than the present method for all Reynolds numbers. This is primarily due to the fact that the flow is not fully developed and hence the local heat transfer coefficient is a strong function of the distance along the collector. In Hottel's method one is forced to choose a mean heat transfer coefficient and this is much lower than the local values near the entrance and, hence, the efficiency predicted is lower. As the Reynolds number increases the heat transfer coefficient also increases and hence the discrepancy is larger.

APPENDIX A

For the flat plate collector the emission of glass is negligible compared to the solar energy absorption by glass in the visible and near-infra red regions. We can, therefore, assume it to be a cold medium. The radiative flux for a tranluscent cold medium is given by Viskanta and Anderson (1975)

$$q_{R\nu}(\kappa_\nu) = \tau_{1\nu}(\mu_o)\left[1 + \rho_{2\nu}(\bar{\mu}_o)\exp(-\kappa_\nu^o/\bar{\mu}_o)\right]\mu_o i^o_{1c\nu}\exp\left(-\frac{\kappa_\nu}{\bar{\mu}_o}\right)/\beta_\nu(\kappa_\nu^o,\bar{\mu}_o)$$

$$+ 2\left[i^o_{1d\nu}\int_o^1 \tau_{1\nu}(\mu')\exp\left(-\frac{\kappa_\nu}{\mu}\right)\mu' d\mu'/\beta_\nu(\kappa_\nu^o,\mu)\right.$$

$$\left.+ i^o_{2d\nu}\int_o^1 \tau_{2\nu}(\mu')\rho_{1\nu}(\mu)\exp\left(-\frac{\kappa_\nu^o + \kappa_\nu}{\mu}\right)\mu' d\mu'/\beta_\nu(\kappa_\nu^o,\mu)\right]$$

$$- \tau_{2\nu}(\mu_o)\left[1+\rho_{1\nu}(\bar{\mu}_o)\exp(-\kappa_\nu^o/\bar{\mu}_o)\right]\mu_o i^o_{2c\nu}\exp\left(-\frac{\kappa_\nu^o - \kappa_\nu}{\bar{\mu}_o}\right)/\beta_\nu(\kappa_\nu^o,\bar{\mu}_o)$$

$$- 2\left[i^o_{1d\nu}\int_o^1 \tau_{1\nu}(\mu')\rho_{2\nu}(\mu)\exp\left(-\frac{2\kappa_\nu^o - \kappa_\nu}{\mu}\right)\mu' d\mu'/\beta_\nu(\kappa_\nu^o,\mu)\right.$$

$$\left.+ i^o_{2d\nu}\int_o^1 \tau_{2\nu}(\mu')\exp\left(-\frac{\kappa_\nu^o - \kappa_\nu}{\mu}\right)\mu' d\mu'/\beta_\nu(\kappa_\nu^o,\mu)\right] \quad (A.1)$$

where $$\beta_\nu(\kappa_\nu^o,\mu) = 1 - \rho_{1\nu}(\mu)\rho_{2\nu}(\mu)\exp\left(-\frac{2\kappa_\nu^o}{|\mu|}\right) \quad (A.2)$$

$$\mu^2 = 1 - (n_o/n_\nu)^2(1-\mu'^2) \quad (A.3)$$

$$\kappa_\nu = \int_0^y k_\nu(y)\, dy, \quad \kappa_\nu^o = \int_0^b k_\nu(y)\, dy \tag{A.4}$$

and $\bar{\mu}_o$ is the value of μ corresponding to $\mu' = \mu_o$ in Eqn. (A.3).

The first term on the right hand side of Eqn. (A.1) denotes the heat flux due to the collimated beam incident on interface 1 after taking into account multiple internal reflection and attenuation. The first term within the parentheses of the second term represents the heat flux due to the diffuse component incident on interface 1 and the other term takes into account the reflection at interface 1 of the diffuse component of radiation incident at interface 2. The last two terms can be explained similarly. These are, however, negative quantities as they are backward and directed.

For the case of a flat plate collector, only the first term and the first term within the parentheses of the second term on the right hand side of Eqn. (A.1) are of importance as all the other terms are negligible as compared to these two. The intensity of radiation at interface 2 of the glass plate is diffuse in nature and does not contain a collimated component. Moreover, the radiant energy from the collector lying in the regions in which glass is transparent to radiation is negligible. Hence, the contribution from the collector plate to the radiative heat flux within the glass plate will be negligible as compared to that from the solar energy. Eqn. (A.1), therefore, reduces to

$$q_{R\nu}(\kappa_\nu) = \tau_{1\nu}(\mu_o) \left[1 + \rho_{2\nu}(\bar{\mu}_o) \exp\left(-\frac{\kappa_\nu^o}{\bar{\mu}_o}\right)\right] \mu_o \, i_{1c\nu}^o \exp\left(-\frac{\kappa_\nu}{\bar{\mu}_o}\right) / \beta_\nu(\kappa_\nu^o, \bar{\mu}_o)$$

$$+ 2\, i_{1d\nu}^o \int_0^1 \tau_{1\nu}(\mu') \exp(-\kappa_\nu/\mu)\, \mu' \, d\mu' / \beta_\nu(\kappa_\nu^o, \mu) \qquad (A.5)$$

Eqn. (A.5) gives the radiative heat flux within the glass plate in a small frequency interval around the frequency ν. In order to obtain the total radiative heat flux within the glass plate, one has to integrate Eqn. (A.5) over the entire range of frequency.

APPENDIX B

The reflectivity in the transparent region [see Siegel and Howell (1972)] may be expressed as

$$\rho_\nu(\mu') = \frac{1}{2}\ \frac{\sin^2(\theta' - \theta)}{\sin^2(\theta' + \theta)} \left[1 + \frac{\cos^2(\theta' + \theta)}{\cos^2(\theta' - \theta)}\right] \tag{B.1}$$

We also have the following relations :

$$n_\nu = \sin\theta'/\sin\theta\ ,\quad \mu' = \cos\theta',\ \text{and}\ \mu = \cos\theta \tag{B.2}$$

From these relations we get

$$\mu^2 = 1-(1-\mu'^2)/n_\nu^2 \tag{B.3}$$

$$\tan\theta' = \{1-\mu'^2\}^{.5}/\mu' \ \text{and}\ \tan\theta = \{1-\mu'^2\}^{.5}/\{n_\nu^2 - (1-\mu'^2)\}^{.5} \tag{B.4}$$

Also

$$\frac{\cos(\theta' + \theta)}{\cos(\theta' - \theta)} = \frac{1 - \tan\theta'\ \tan\theta}{1 + \tan\theta'\ \tan\theta}$$

or

$$\frac{\cos(\theta' + \theta)}{\cos(\theta' - \theta)} = \frac{\mu'\ \{n_\nu^2 - (1-\mu'^2)\}^{.5} - (1-\mu'^2)}{\mu'\ \{n_\nu^2 - (1-\mu'^2)\}^{.5} + (1-\mu'^2)} \tag{B.5}$$

and

$$\frac{\sin(\theta' - \theta)}{\sin(\theta' + \theta)} = \frac{1 - \cot\theta'\ \tan\theta}{1 + \cot\theta'\ \tan\theta}$$

or

$$\frac{\sin(\theta' - \theta)}{\sin(\theta' + \theta)} = \frac{\{n_\nu^2 - (1-\mu'^2)\}^{.5} - \mu'}{\{n_\nu^2 - (1-\mu'^2)\}^{.5} + \mu'} \tag{B.6}$$

Substitution of Eqns. (B.3), (B.5) and (B.6) into Eqn. (B.1) results in

$$\rho_\nu(\mu') = 1 - 2\mu' \{n_\nu^2 - (1 - \mu'^2)\}^{.5}\left[\left(\frac{1}{\mu' + \{n_\nu^2 - (1 - \mu'^2)\}^{.5}}\right)^2\right.$$

$$\left. + \left(\frac{n_\nu}{\mu' n_\nu^2 + \{n_\nu^2 - (1 - \mu'^2)\}^{.5}}\right)^2\right] \tag{B.7}$$

and $$\rho_\nu(\mu) = 1 - 2n_\nu \mu \{1 - n_\nu^2 (1 - \mu^2)\}^{.5}\left[\left(\frac{1}{n_\nu \mu + \{1 - n_\nu^2 (1 - \mu^2)\}^{.5}}\right)^2\right.$$

$$\left. + \left(\frac{1}{\mu + n_\nu \{1 - n_\nu^2 (1 - \mu^2)\}^{.5}}\right)^2\right] \tag{B.8}$$

The transmittivities are then given by

$$\tau_\nu(\mu') = 1 - \rho_\nu(\mu') \tag{B.9}$$

and $$\tau_\nu(\mu) = 1 - \rho_\nu(\mu) \tag{B.10}$$

The effective directional reflectance $r_\nu(\mu')$ and transmittance, $t_\nu(\mu')$ are given below [Viskanta (1975)]

$$r_\nu(\mu') = \rho_{1\nu}(\mu') + \rho_{2\nu}(\mu)\tau_{1\nu}(\mu')\tau_{2\nu}(\mu) \exp(-2 k_\nu b/\mu)/\beta_\nu(k_\nu b, \mu) \tag{B.11}$$

and $$t_\nu(\mu') = \tau_{1\nu}(\mu') \tau_{2\nu}(\mu) \exp(-2k_\nu b/\mu)/\beta_\nu(k_\nu b, \mu) \tag{B.12}$$

The hemispherical reflectance and transmittance for diffuse radiation incident on the semitransparent sheet is given by

$$r_\nu = 2 \int_0^1 r_\nu(\mu') \mu' d\mu' \tag{B.13}$$

$$t_\nu = 2 \int_0^1 t_\nu(\mu') \mu' d\mu' \tag{B.14}$$

APPENDIX C

1) Energy balance for the glass plate.

The expressions for $B_{1\nu}$, $H_{1\nu}$, $B_{2\nu}$ and $B_{3\nu}$ are from Chapter 3

$$B_{1\nu} = \varepsilon_{g\nu}\, e_{b\nu}(T_g) + r_{gc\nu}(\mu')\, i^{o}_{1c\nu} + \pi\, r_{gd\nu}\, i^{o}_{1d\nu} + e_{b\nu}(T_g)\, r_{gd\nu} + t_{gd\nu}\, B_{3\nu} \tag{3.2}$$

$$H_{1\nu} = i^{o}_{1c\nu} + \pi\, i^{o}_{1d\nu} + e_{b\nu}(T_\infty) \tag{3.1}$$

$$B_{2\nu} = \{t_{gc\nu}(\mu')\, i^{o}_{1c\nu} + \pi\, t_{gd\nu}\, i^{o}_{1d\nu} + t_{gd\nu}\, e_{b\nu}(T_\infty) + \varepsilon_{g\nu}\, e_{b\nu}(T_g) + r_{gd\nu}\, \varepsilon_{3\nu}\, e_{b\nu}(T_3)\}/\{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})\} \tag{3.6}$$

$$B_{3\nu} = \{\varepsilon_{3\nu}\, e_{b\nu}(T_3) + (1 - \varepsilon_{3\nu})\, [\varepsilon_{g\nu}\, e_{b\nu}(T_g) + t_{gc\nu}(\mu')\, i^{o}_{1c\nu} + \pi\, t_{gd\nu}\, i^{o}_{1d\nu} + t_{gd\nu}\, e_{b\nu}(T_\infty)]\}/\{1 - r_{gd\nu}(1-\varepsilon_{3\nu})\} \tag{3.7}$$

Also

$$H_{2\nu} = B_{3\nu} \quad \text{and} \quad H_{3\nu} = B_{2\nu} \tag{3.5}$$

On substitution of Eqn. (3.5) into Eqn. (3.2) we have

$$\begin{aligned} B_{1\nu} = {} & r_{gc\nu}(\mu')\, i^{o}_{1c\nu} + \pi\, r_{gd\nu}\, i^{o}_{1d\nu} + r_{gd\nu}\, e_{b\nu}(T_g) + \varepsilon_{g\nu}\, e_{b\nu}(T_g) \\ & + \frac{t_{gd\nu}(1-\varepsilon_{3\nu})\, t_{gc\nu}(\mu')\, i^{o}_{1c\nu}}{1 - r_{gd\nu}(1 - \varepsilon_{3\nu})} + \pi\, \frac{t_{gd\nu}(1-\varepsilon_{3\nu})\, t_{gd\nu}}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}\, i^{o}_{1d\nu} \\ & + \frac{t_{gd\nu}(1-\varepsilon_{3\nu})\, t_{gd\nu}}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}\, e_{b\nu}(T_\infty) + \frac{\varepsilon_{g\nu}(1 - \varepsilon_{3\nu})\, t_{gd\nu}}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}\, e_{b\nu}(T_g) \\ & + \frac{t_{gd\nu}\, \varepsilon_{3\nu}\, e_{b\nu}(T_3)}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})} \end{aligned} \tag{C.1}$$

Also

$$
\begin{aligned}
B_1 &= \int_0^\infty B_{1\nu}\, d\nu \\
B_2 &= \int_0^\infty B_{2\nu}\, d\nu \\
H_1 &= \int_0^\infty H_{1\nu}\, d\nu \\
H_2 &= \int_0^\infty H_{2\nu}\, d\nu
\end{aligned}
\tag{C.2}
$$

$$
\therefore\ H_1 - B_1 + H_2 - B_2 = \int_0^\infty i^o_{1c\nu}[1 - r_{gc\nu}(\mu')
$$

$$
- \frac{t_{gd\nu}(1-\varepsilon_{3\nu})t_{gc\nu}(\mu') - (1-\varepsilon_{3\nu})t_{gc\nu}(\mu') + t_{gc\nu}(\mu')}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}]\, d\nu
$$

$$
+ \int_0^\infty (\pi i^o_{1d\nu} + e_{b\nu}(T_\infty))\ [1 - r_{gd\,\nu}
$$

$$
- \frac{t_{gd\nu}(1- \ _{3\nu})t_{gd\nu} - (1-\varepsilon_{3\nu})\ t_{gd\nu} + t_{gd\nu}}{1 - r_{gd\nu}\ (1-\varepsilon_{3\nu})}]d\nu
$$

$$
+ \int_0^\infty \varepsilon_{g\nu}\ e_{b\nu}(T_g)\ [-1 - \frac{t_{gd\nu}(1-\varepsilon_{3\nu}) - (1-\varepsilon_{3\nu}) + 1}{1 - r_{gd\,\nu}(1-\varepsilon_{3\,\nu})}]\, d\nu
$$

$$
+ \int_0^\infty \varepsilon_{3\nu}\ e_{b\nu}(T_3)\ \ [- \frac{t_{gd\nu} - 1 + r_{gd\nu}}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}]d\nu
$$

$$
= \int_0^\infty i^o_{1c\nu}[1 - r_{gc\nu}(\mu')]\, d\nu + \int_0^\infty i^o_{1c\nu}\ t_{gc\nu}(\mu') \times
$$

$$
[\frac{(1-\varepsilon_{3\nu})\ (1 - t_{gd\nu}) - 1}{1 - r_{gd\nu}\ (1-\varepsilon_{3\nu})}]d\nu
$$

$$+ \int_0^\infty (1-r_{gd\nu})(\pi\, i^0_{1d\nu} + e_{b\nu}(T_\infty))\, d\nu + \int_0^\infty t_{gd\nu}\, [i^0_{1d\nu} + e_{b\nu}(T_\infty)] \times$$

$$[\frac{(1-\varepsilon_{3\nu})\,(1-t_{gd\nu}) - 1}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}]\, d\nu$$

$$+ \int_0^\infty \varepsilon_{g\nu}\, e_{b\nu}(T_g)\ [\frac{(1-\varepsilon_{3\nu})(1-t_{gd\nu}) - 1}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})} - 1]\ d\nu$$

$$+ \int_0^\infty \frac{\varepsilon_{3\nu} e_{b\nu}(T_3)\ [1 - r_{gd\nu} - t_{gd\nu}]}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}\, d\nu$$

$$= \int_0^\infty i^0_{1c\nu}\, [1 - r_{gc\nu}(\mu')]\, d\nu + \int_0^\infty t_{gc\nu}(\mu')\ i^0_{1c\nu}\ \frac{(1-\varepsilon_{3\nu})(1-t_{gd\nu})-1}{1-r_{gd\nu}(1-\varepsilon_{3\nu})}\ d\nu$$

$$+ \pi \int_0^\infty (1-r_{gd\nu})\, i^0_{1d\nu}\, d\nu + \pi \int_0^\infty t_{gd\nu}\, i^0_{1d\nu}\ \frac{(1-\varepsilon_{3\nu})(1-t_{gd\nu})-1}{1-r_{gd\nu}(1-\varepsilon_{3\nu})}\ d\nu$$

$$+ \int_0^\infty (1-r_{gd\nu}-t_{gd\nu})\, e_{b\nu}(T_\infty)\, d\nu + \int_0^\infty t_{gd\nu}\ e_{b\nu}(T_\infty) \times$$

$$[\frac{(1-\varepsilon_{3\nu})(1-t_{gd\nu}) - 1}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})} + 1]\ d\nu$$

$$+ \int_0^\infty \varepsilon_{g\nu}\ e_{b\nu}(T_g)\ \frac{(1-\varepsilon_{3\nu})(1-t_{gd\nu})-1+\varepsilon_{3\nu}}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}\ d\nu$$

$$- \int_0^\infty \frac{\varepsilon_{3\nu}\ \varepsilon_{g\nu}\ e_{b\nu}(T_g)}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}\, d\nu - \int_0^\infty \varepsilon_{g\nu}\ e_{b\nu}(T_g)\, d\nu$$

$$+ \int_0^\infty \frac{\varepsilon_{3\nu}\ \varepsilon_{g\nu}\ e_{b\nu}(T_3)}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}\, d\nu$$

$$\therefore\ (H_1-B_1)+(H_2-B_2) = \int_0^\infty I^o_{1c\nu}\,[1-r_{gc\nu}(\mu')\,]\,d\nu + \int_0^\infty t_{gc\nu}(\mu')\, I^o_{1c\nu} \times$$

$$\left[\frac{(1-\varepsilon_{3\nu})(1-t_{gd\nu})-1}{1-r_{gd\nu}(1-\varepsilon_{3\nu})}\right] d\nu \quad + \pi \int_0^\infty I^o_{1d\nu}\,[1-r_{gd\nu}]\,d\nu$$

$$+ \pi \int_0^\infty t_{gd\nu}\, I^o_{1d\nu}\left[\frac{(1-\varepsilon_3)\,(1-t_{gd\nu})-1}{1-r_{gd\nu}(1-\varepsilon_{3\nu})}\right] d\nu$$

$$+ \int_0^\infty \varepsilon_{g\nu}\,[e_{b\nu}(T_\infty)-e_{b\nu}(T_g)\,]\,d\nu + \int_0^\infty \frac{\varepsilon_{3\nu}\,\varepsilon_{g\nu}\,[e_{b\nu}(T_3)-e_{b\nu}(T_g)\,]}{1-r_{gd\nu}(1-\varepsilon_{3\nu})}\,d\nu$$

$$+ \int_0^\infty \frac{t_{gd\nu}(1-\varepsilon_{3\nu})\;\varepsilon_{g\nu}\;[e_{b\nu}(T_\infty)-e_{b\nu}(T_g)]}{1-r_{gd\nu}\,(1-\varepsilon_{3\nu})}\,d\nu \qquad (C.3)$$

Eqn. (C.3) may be further reduced to

$$(H_1-B_1)+(H_2-B_2) = \int_0^\infty I^o_{1c\nu}\,[\,1-r_{gc\nu}(\mu')\,-t_{gc\nu}(\mu')\,]\;d\nu + \int_0^\infty t_{gc\nu}(\mu')\,I^o_{1c\nu}\times$$

$$\left[\frac{(1-\varepsilon_{3\nu})\,(1-t_{gd\nu})-1}{1-r_{gd\nu}(1-\varepsilon_{3\nu})}+1\right] d\nu$$

$$+ \pi \int_0^\infty I^o_{1d\nu}\,[1-r_{gd\nu}-t_{gd\nu}\,]\,d\nu + \pi \int_0^\infty t_{gd\nu}\, I^o_{1d\nu} \times$$

$$\left[\frac{(1-\varepsilon_{3\nu})\,(1-t_{gd\nu})-1}{1-r_{gd\nu}(1-\varepsilon_{3\nu})}+1\right] d\nu$$

$$+ \int_0^\infty \varepsilon_{g\nu}\,[\,e_{b\nu}(T_\infty)-e_{b\nu}(T_g)\,]\,d\nu$$

$$+ \int_0^\infty \frac{\varepsilon_{3\nu}\,\varepsilon_{g\nu}\,[e_{b\nu}(T_3)-e_{b\nu}(T_g)]}{1-r_{gd\nu}\,(1-\varepsilon_{3\nu})}\,d\nu$$

$$+ \int_0^\infty \frac{t_{gd\nu}\,(1-\varepsilon_{3\nu})\;\varepsilon_{g\nu}\;[e_{b\nu}(T_\infty) - e_{b\nu}(T_g)]}{1 - r_{gd\nu}\,(1-\varepsilon_{3\nu})}\,d\nu$$

$$\text{or } (H_1-B_1)+(H_2-B_2) = \int_0^\infty \varepsilon_{g\nu}\;[e_{b\nu}(T_\infty) - e_{b\nu}(T_g)]\;d\nu$$

$$+ \int_0^\infty \frac{\varepsilon_{3\nu}\;\varepsilon_{g\nu}\;[e_{b\nu}(T_3) - e_{b\nu}(T_g)]}{1 - r_{gd\nu}\,(1-\varepsilon_{3\nu})}\,d\nu$$

$$+ \int_0^\infty \varepsilon_{g\nu}(\mu')\; i^o_{1c\nu}\;d\nu + \pi\int_0^\infty \varepsilon_{g\nu}\; i^o_{1d\nu}\;d\nu$$

$$+ \int_0^\infty \frac{t_{gc\nu}(\mu')(1-\varepsilon_{3\nu})\varepsilon_{g\nu}\; i^o_{1c\nu}}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}\,d\nu$$

$$+ \pi\int_0^\infty \frac{t_{gd\nu}\,(1-\varepsilon_{3\nu})\;\varepsilon_{g\nu}\, i^o_{1d\nu}}{1 - r_{gd\nu}\,(1-\varepsilon_{3\nu})}\,d\nu$$

$$+ \int_0^\infty \frac{t_{gd\nu}(1-\varepsilon_{3\nu})\varepsilon_{g\nu}\,[e_{b\nu}(T_\infty) - e_{b\nu}(T_g)]}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}\,d\nu \qquad (C.4)$$

2) <u>Energy Balance for the collector plate</u>

From Eqns. (3.5) and (3.7) we have

$$H_3-B_3 = \int_0^\infty \frac{t_{gc\nu}(\mu')\;\varepsilon_{3\nu}\; i^o_{1c\nu}}{1-r_{gd\nu}\,(1-\varepsilon_{3\nu})}\,d\nu + \pi\int_0^\infty \frac{t_{gd\nu}\;\varepsilon_{3\nu}\, i^o_{1d\nu}}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})}\,d\nu$$

$$+ \int_0^\infty \frac{t_{gd\nu}\;\varepsilon_{3\nu}\;e_{b\nu}(T_\infty)}{1 - r_{gd\nu}\,(1-\varepsilon_{3\nu})}\,d\nu + \int_0^\infty \frac{\varepsilon_{3\nu}\;\varepsilon_{g\nu}\;e_{b\nu}(T_g)}{1-r_{gd\nu}(1-\varepsilon_{3\nu})}\,d\nu$$

$$- \int_0^\infty \frac{\varepsilon_{3\nu} (1 - r_{gd\nu})\, e_{b\nu} (T_3)}{1 - r_{gd\nu} (1 - \varepsilon_{3\nu})} d\nu$$

$$= \int_0^\infty \frac{t_{gc\nu}(\mu')\, \varepsilon_{3\nu}\, i^o_{1c\nu}}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})} d\nu + \pi \int_0^\infty \frac{t_{gd\nu}\, \varepsilon_{3\nu}\, i^o_{1d\nu}}{1-r_{gd\nu}(1-\varepsilon_{3\nu})} d\nu$$

$$+ \int_0^\infty \frac{t_{gd}\, \varepsilon_{3\nu} \{ e_{b\nu}(T_\infty) - e_{b\nu}(T_3) \}}{1 - r_{gd\nu} (1 - \varepsilon_{3\nu})} d\nu$$

$$+ \int_0^\infty \frac{\varepsilon_{3\nu} [\varepsilon_{g\nu}\, e_{b\nu}(T_g) - (1 - r_{gd\nu} - t_{gd\nu})\, e_{b\nu} (T_3)]}{1 - r_{gd\nu} (1 - \varepsilon_{3\nu})} d\nu$$

or $$H_3 - B_3 = \int_0^\infty \frac{t_{gc\nu}(\mu')\, \varepsilon_{3\nu}\, i^o_{1c\nu}}{1 - r_{gd\nu}(1-\varepsilon_{3\nu})} d\nu + \pi \int_0^\infty \frac{t_{gd\nu}\, \varepsilon_{3\nu}\, i^o_{1d\nu}}{1-r_{gd\nu}(1-\varepsilon_{3\nu})} d\nu$$

$$+ \int_0^\infty \frac{t_{gd\nu}\, \varepsilon_{3\nu} \{ e_{b\nu}(T_\infty) - e_{b\nu}(T_3) \}}{1 - r_{gd\nu} (1 - \varepsilon_{3\nu})} d\nu$$

$$+ \int_0^\infty \frac{\varepsilon_{3\nu}\, \varepsilon_{g\nu} [e_{b\nu}(T_g) - e_{b\nu} (T_3)]}{1 - r_{gd\nu} (1 - \varepsilon_{3\nu})} \tag{C.5}$$

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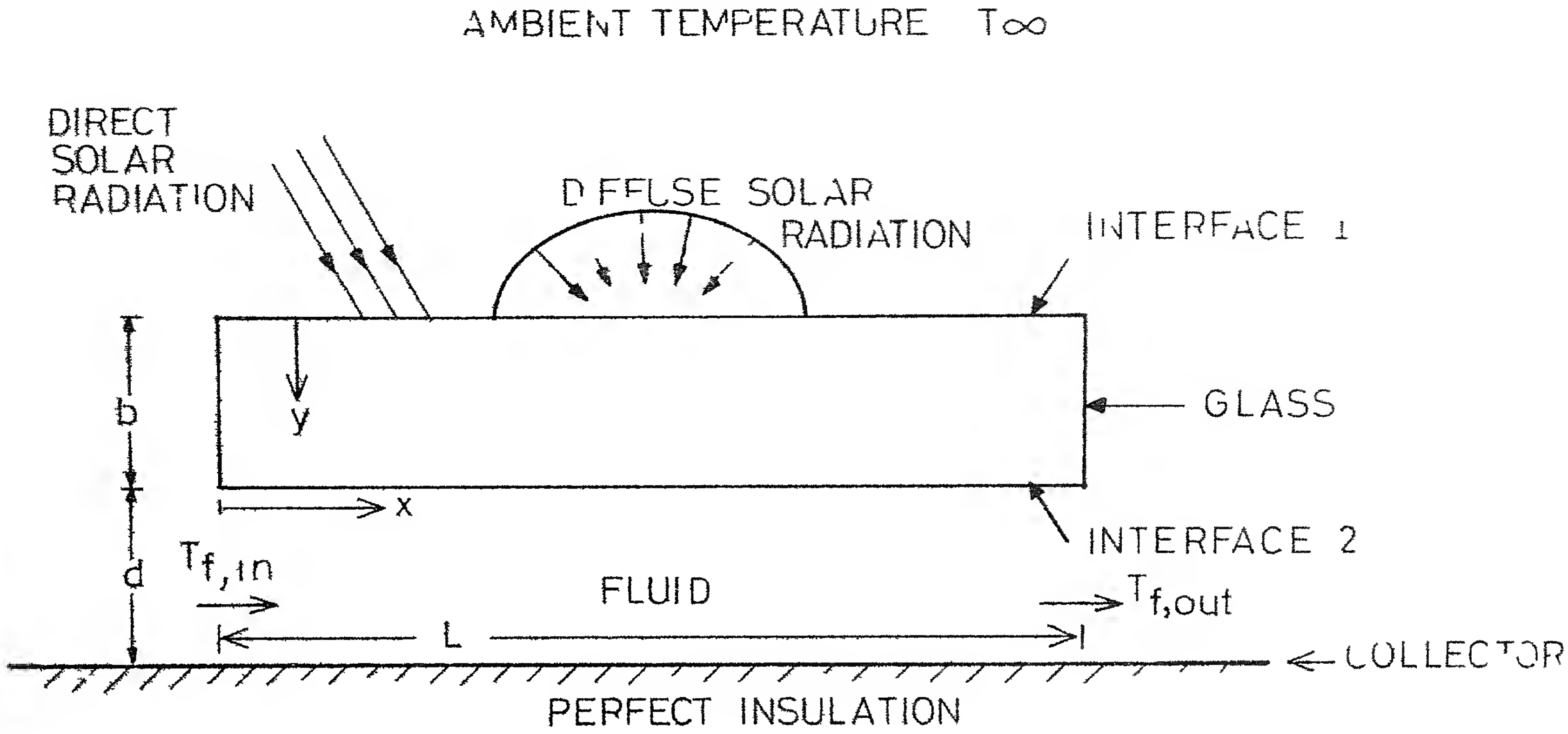

FIG 11 MODEL

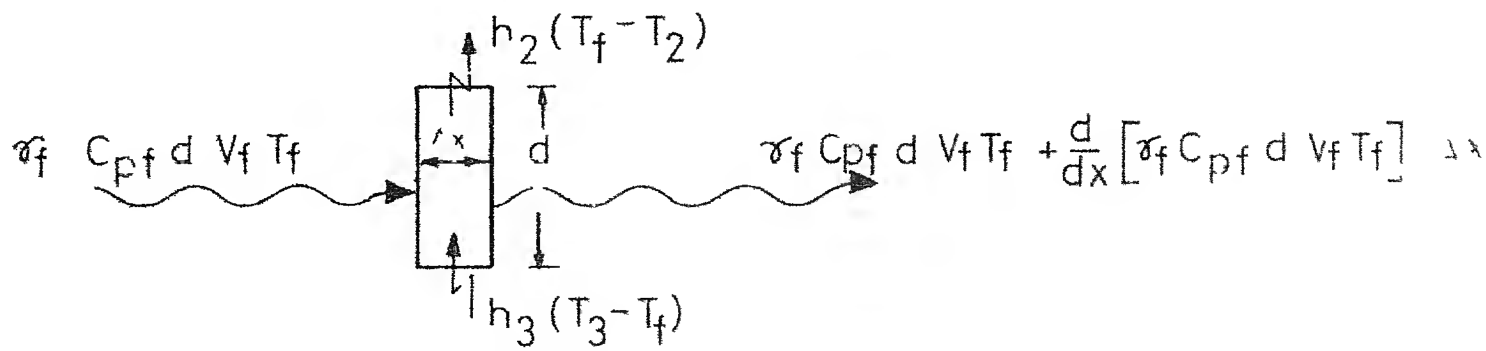

FIG 12 CONTROL VOLUME

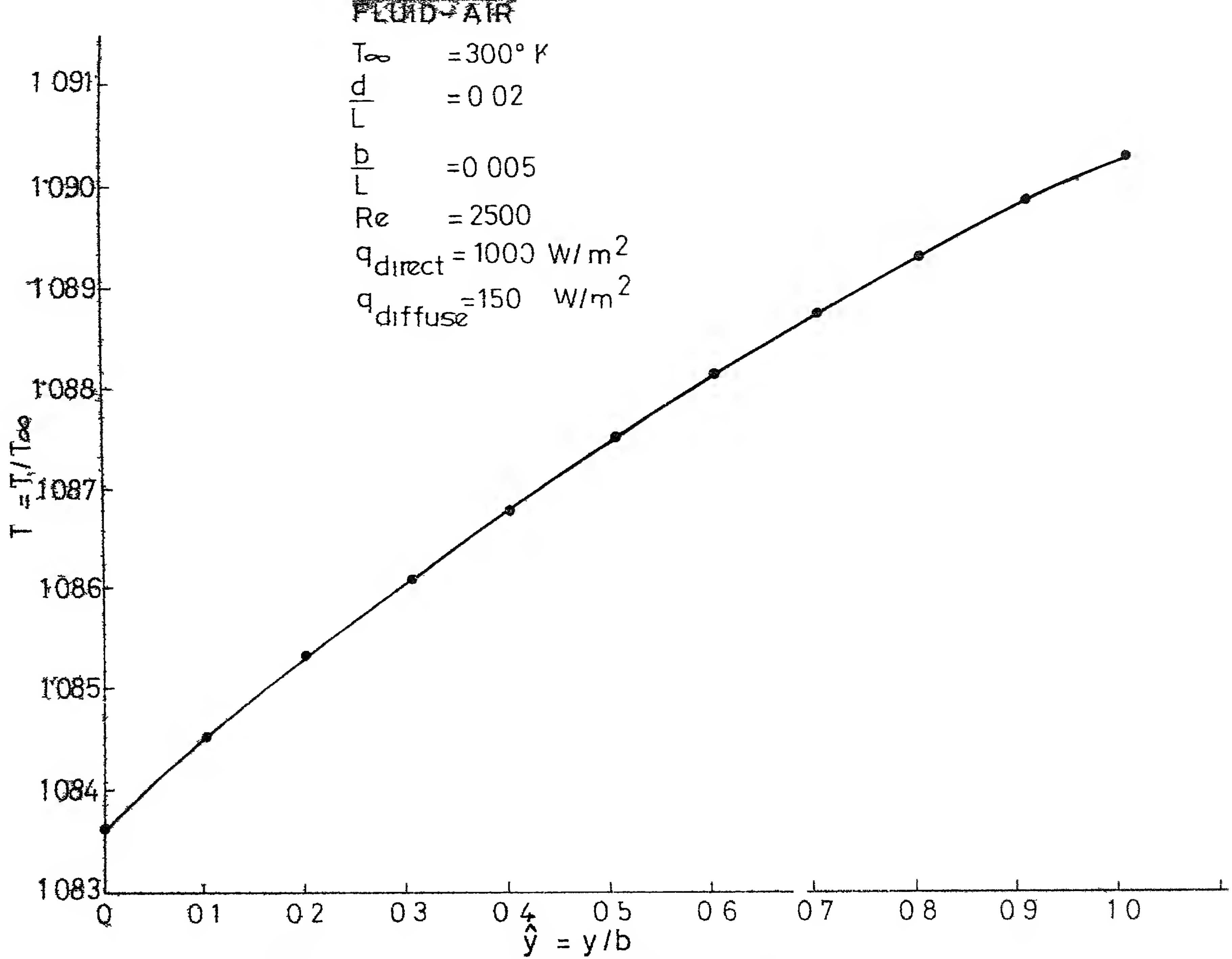

FIG 2·1 TEMPERATURE DISTRIBUTION WITHIN THE GLASS PLATE

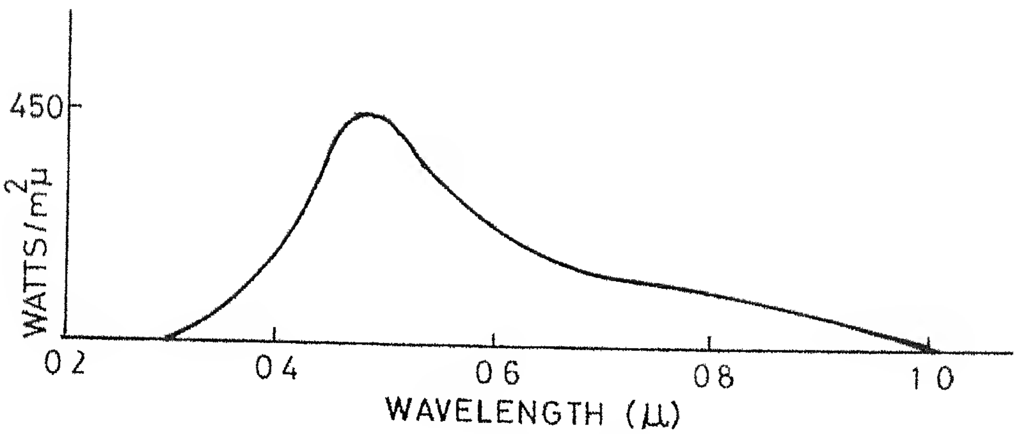

FIG 3 1 SPCTRAL VARIATION OF DIFFUSE SOLAR RADIATION

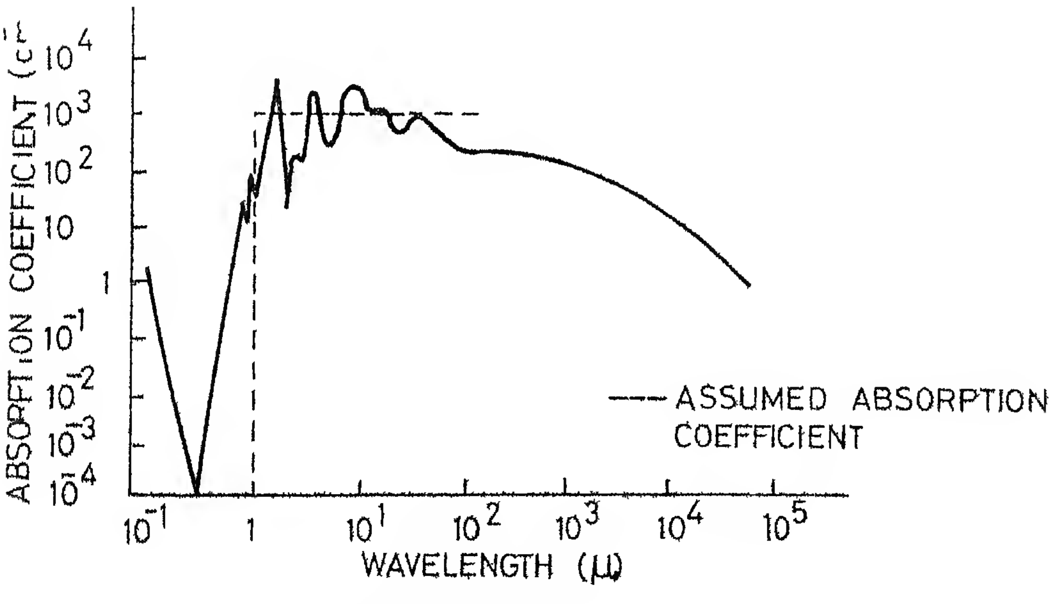

FIG 3 2 ABSORPTION COEFFICIENT SPECTRA OF WATER

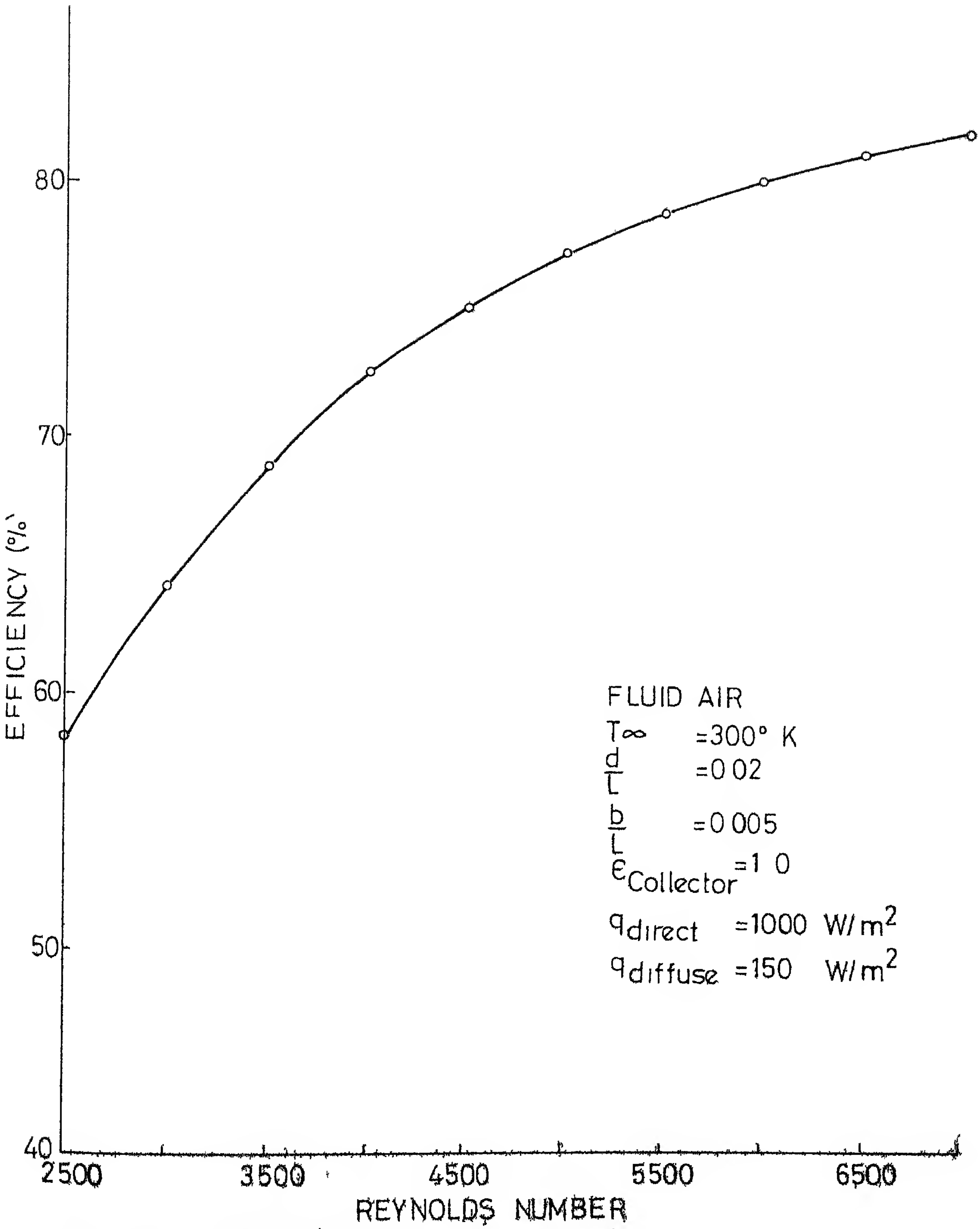

FIG 4·1 EFFICIENY vs FLOW RATE

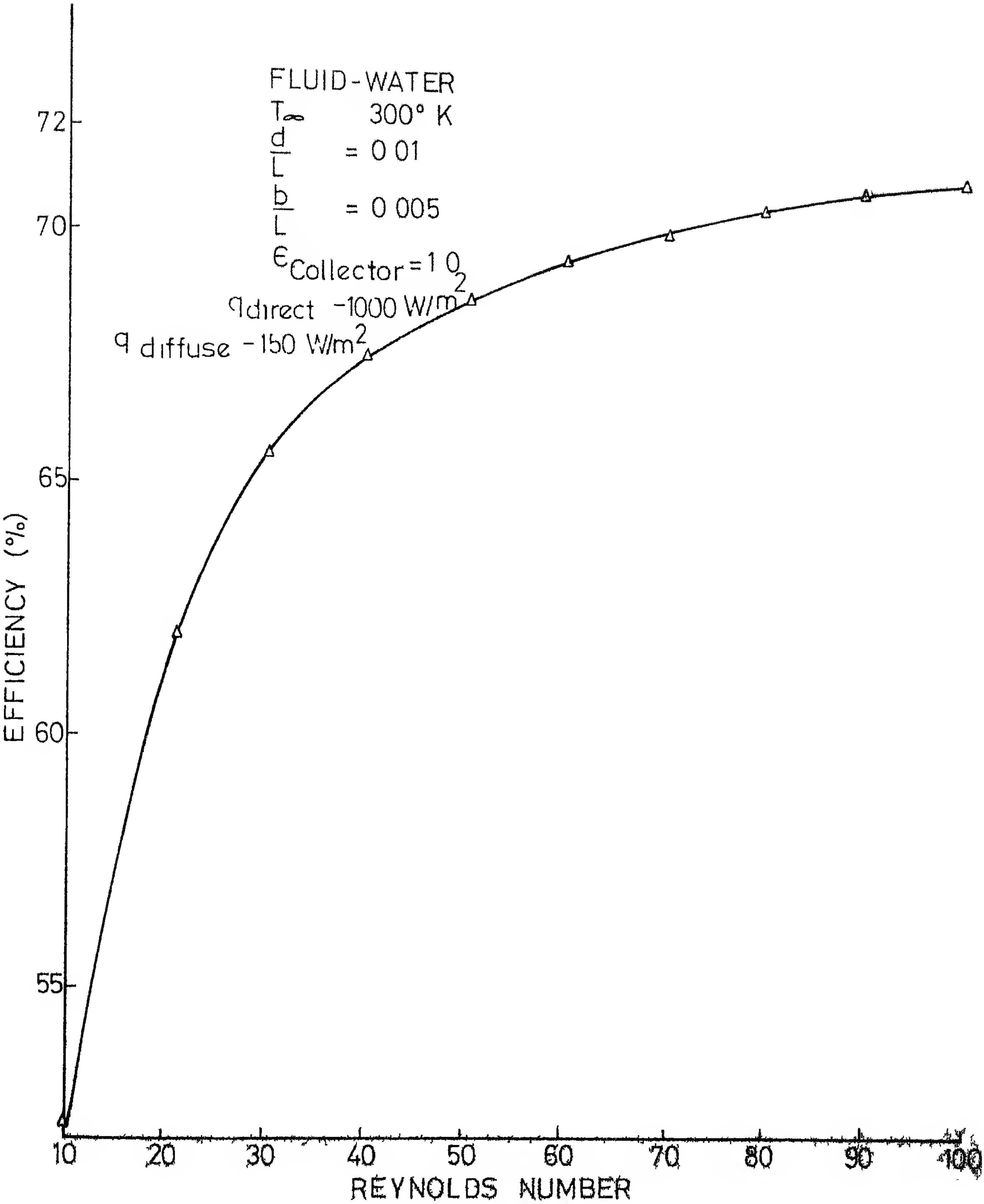

FIG 4·2 EFFICIENCY vs FLOW RATE

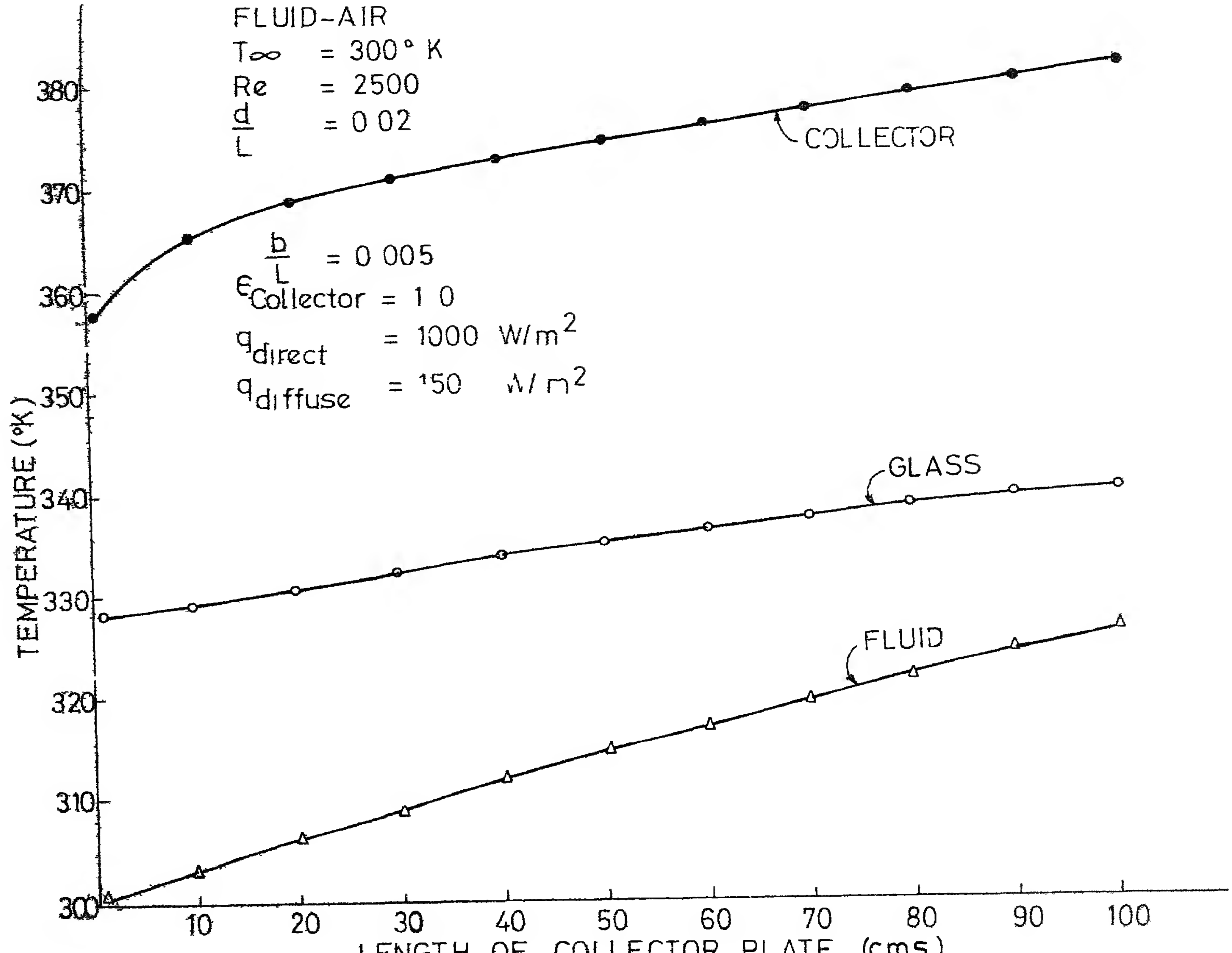

FIG 4 3 VARIATION OF TEMPERATURES

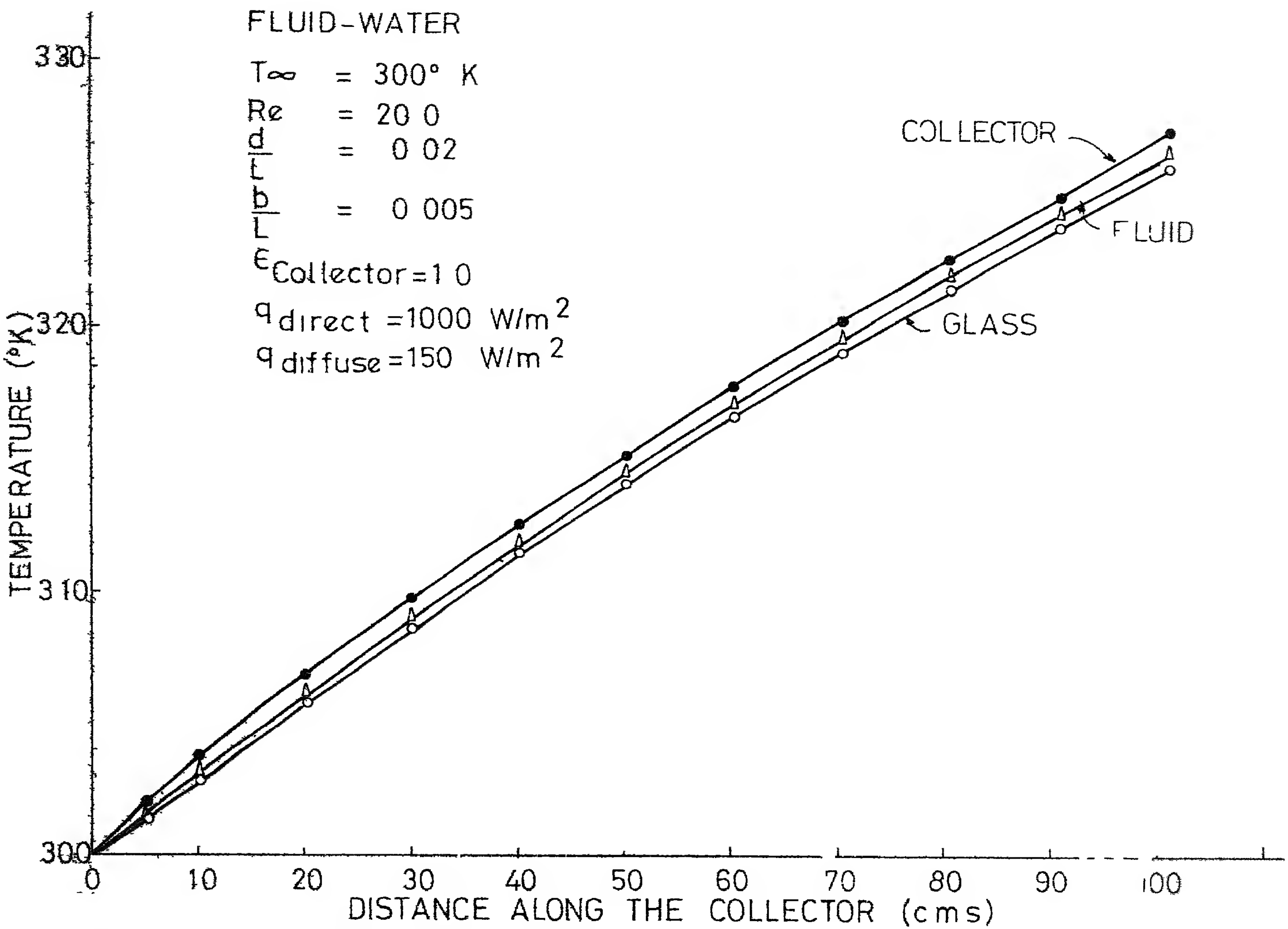

FIG 4 4 VARIATION OF TEMPERATURES

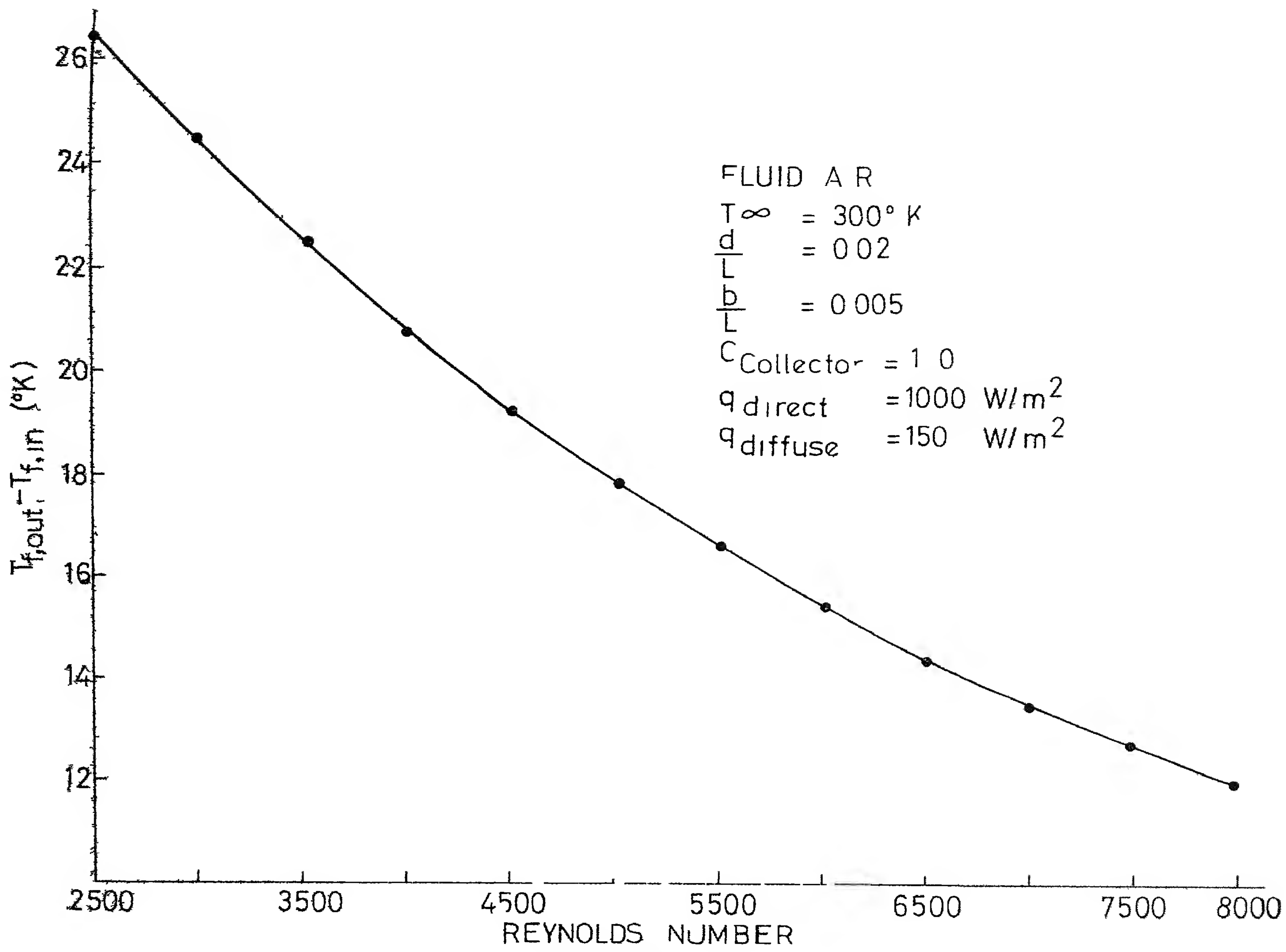

FIG 4·5 TEMPERATURE INCREASE vs FLOW RATE

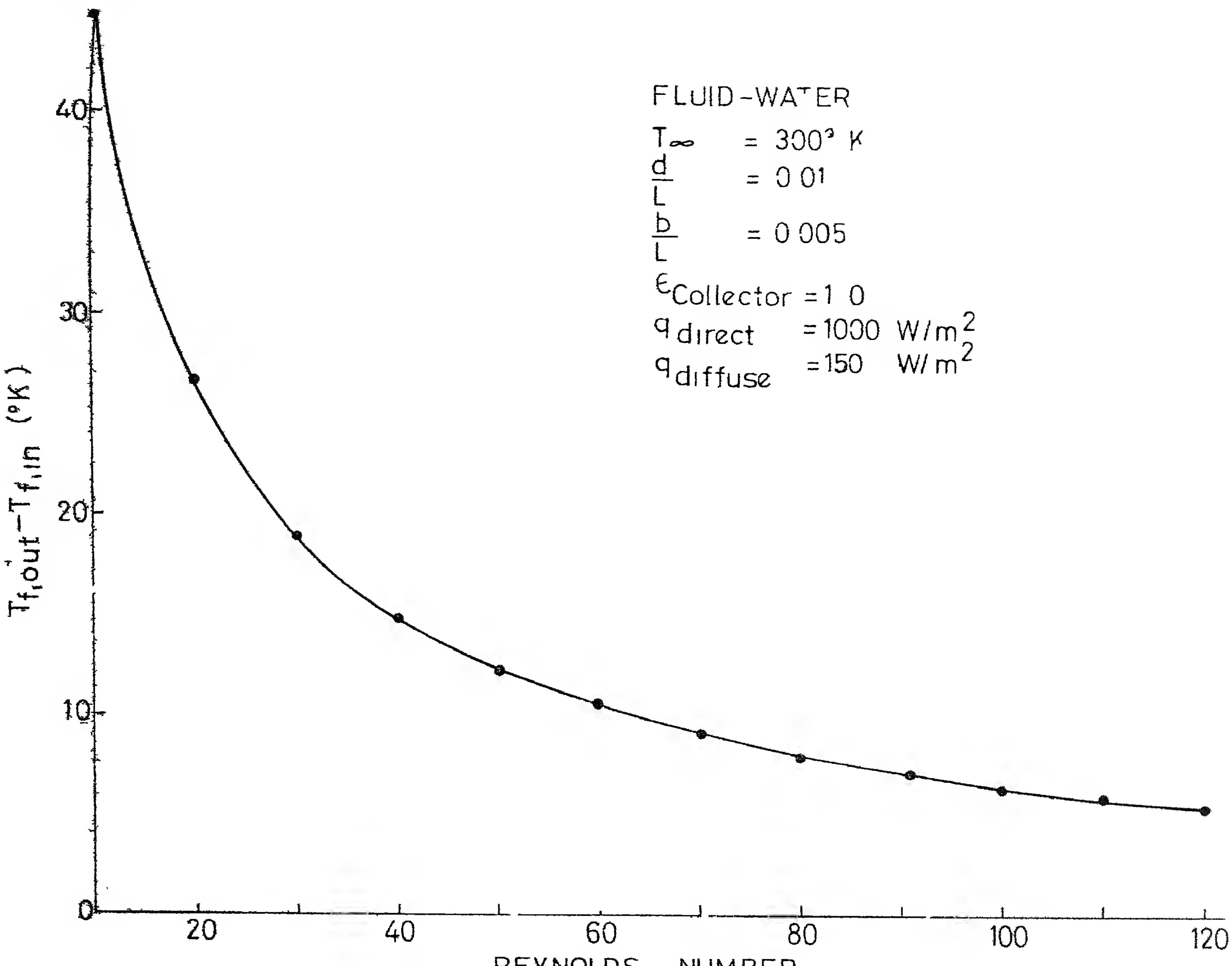

FIG 4 6- TEMPERATURE INCREASE vs FLOW RATE

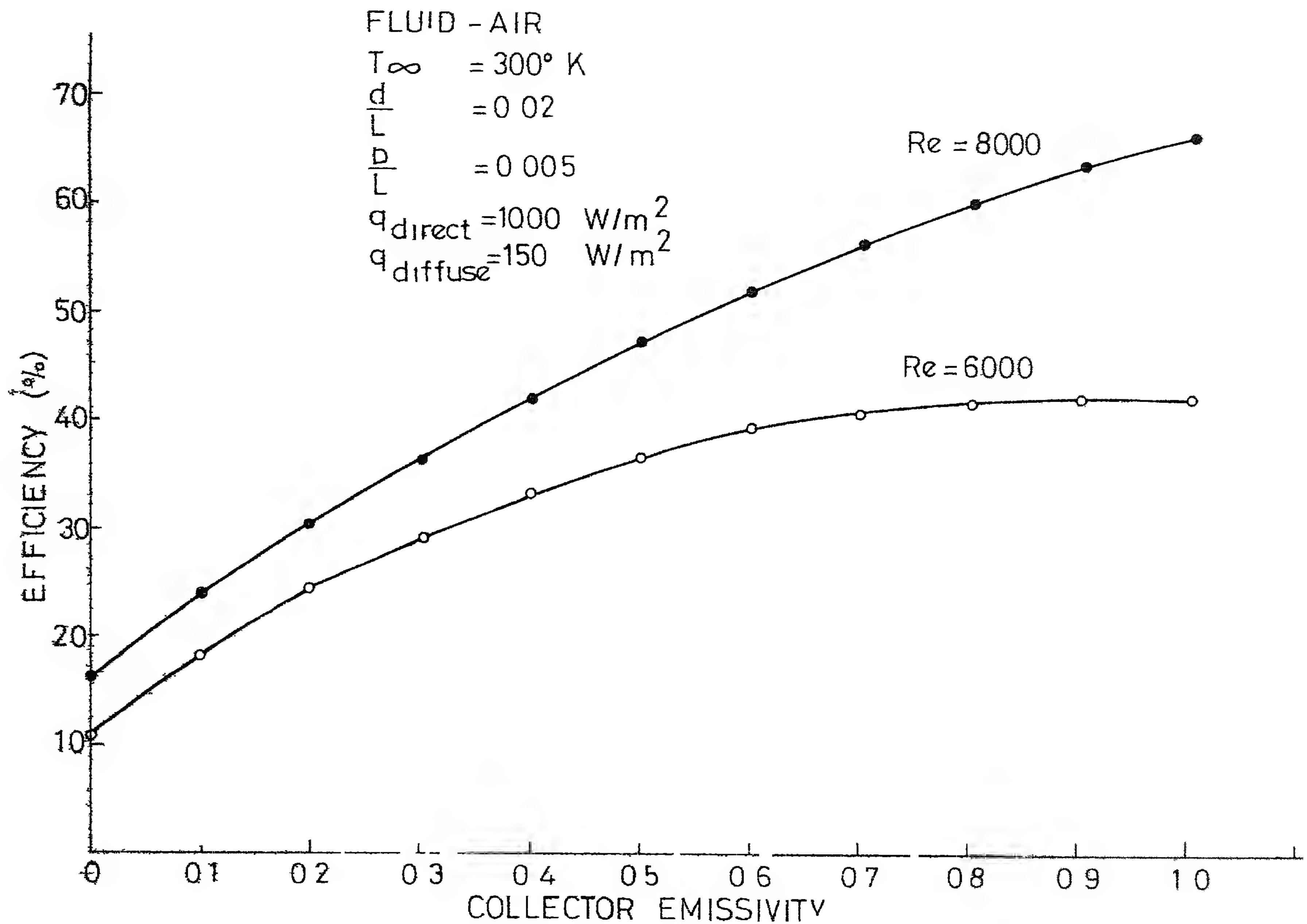

FIG 4 7 EFFICIENCY vs COLLECTOR EMISSIVITY

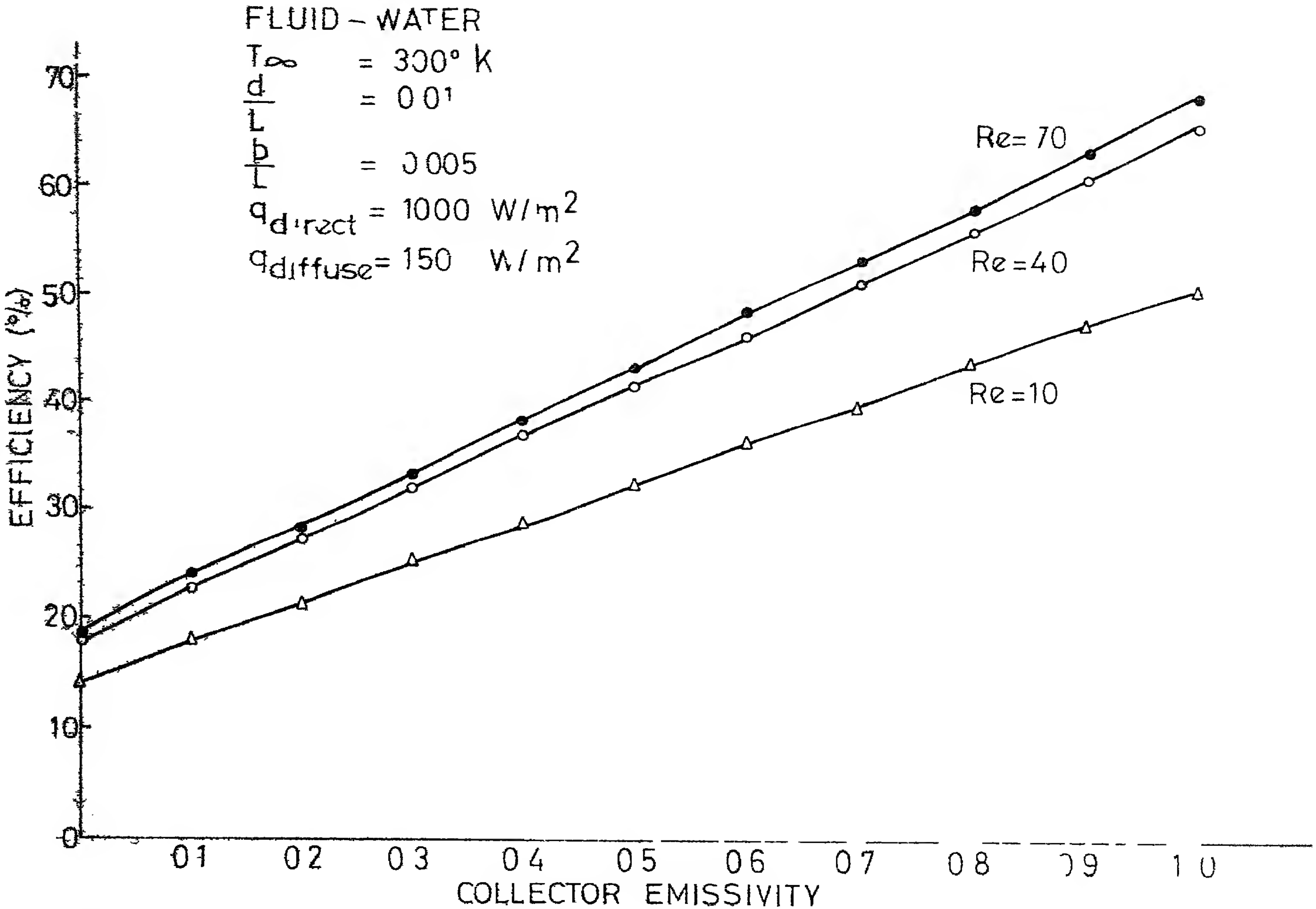

FIG 4 8 COLLECTOR EMISSIVITY EFFECT

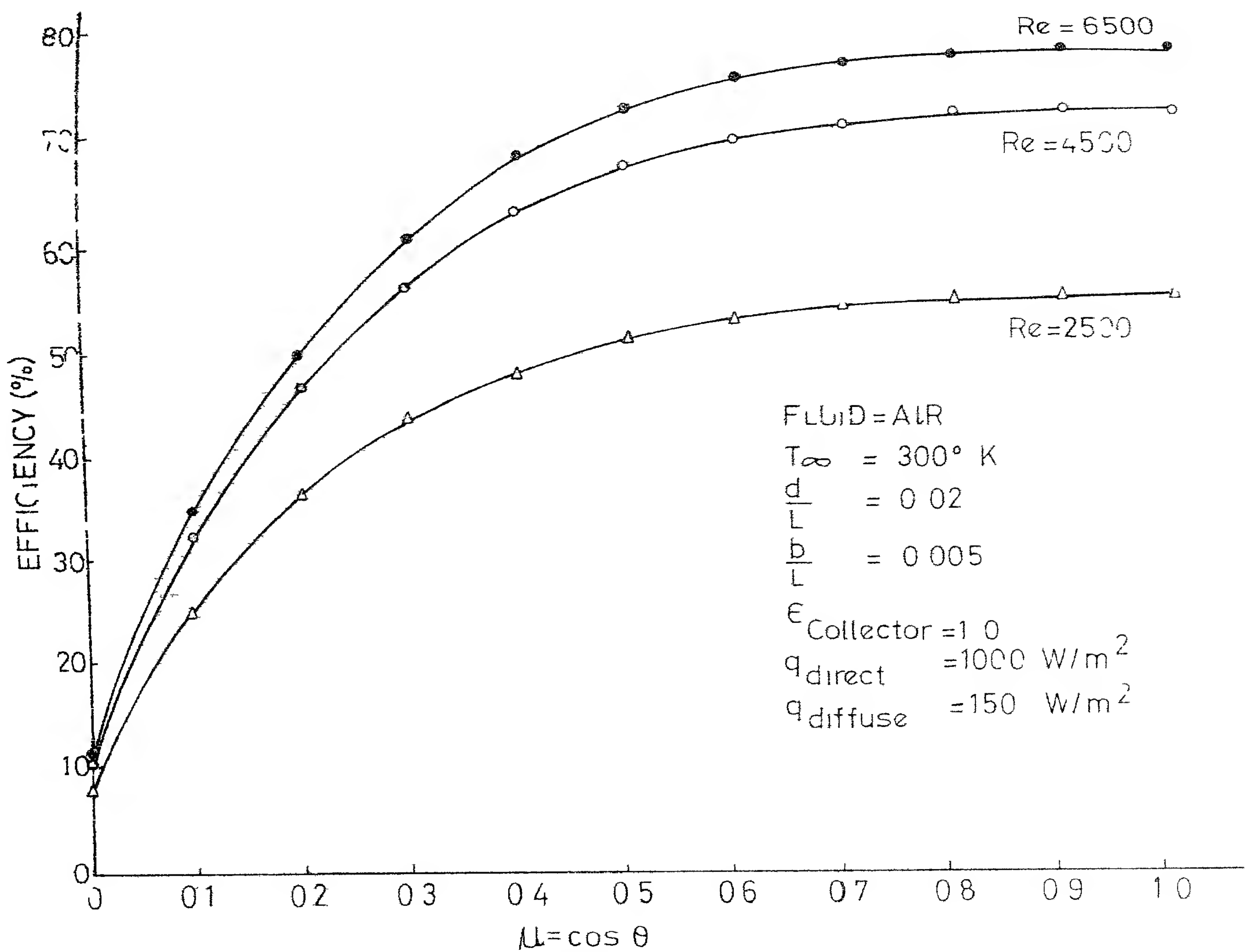

FIG 4 9 EFFICIENCY vs ANGLE OF INCIDENCE OF DIRECT SOLAR RADIATION

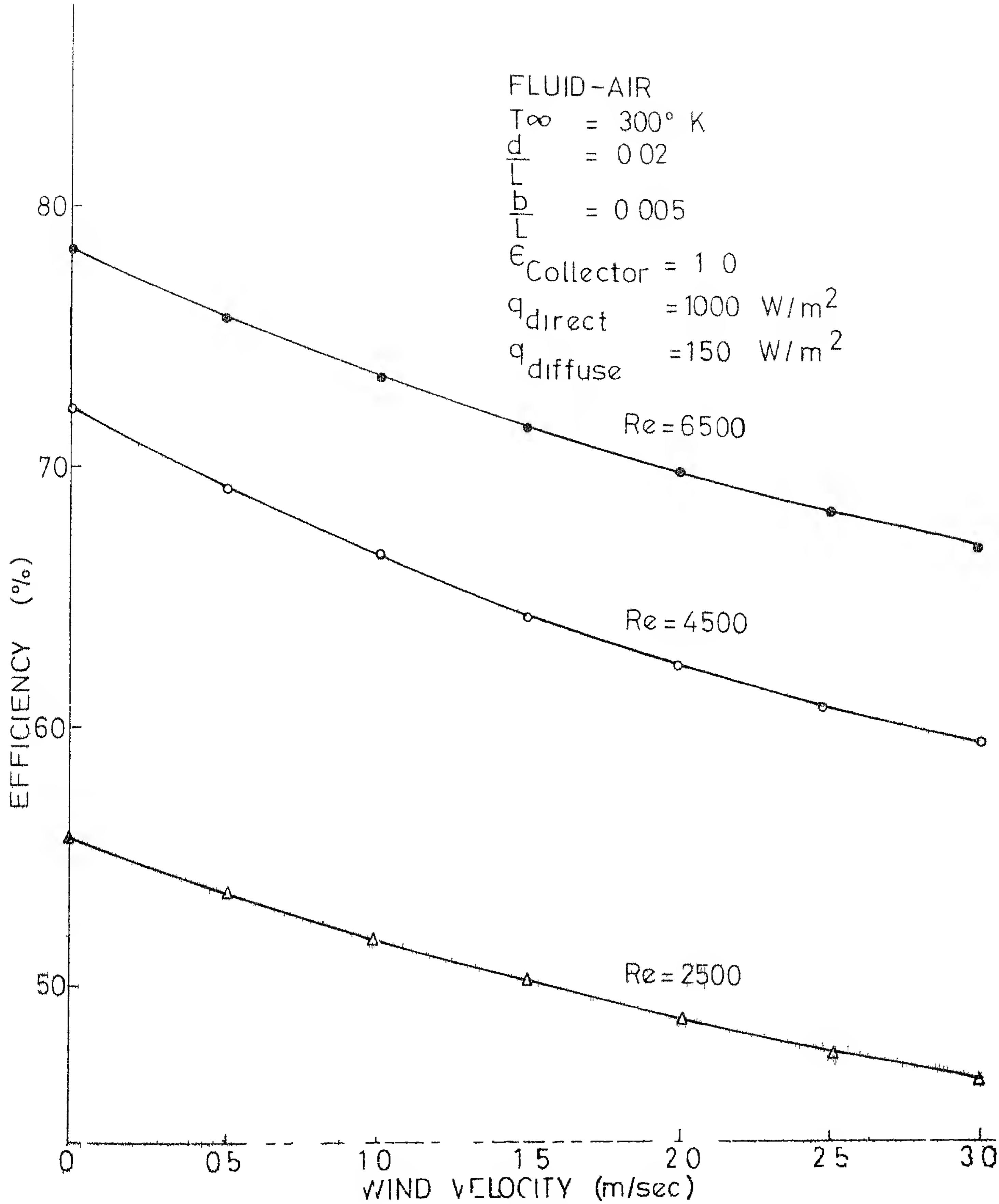

FIG 4 10 EFFICIENCY vs AMBIENT WIND VELOCITY

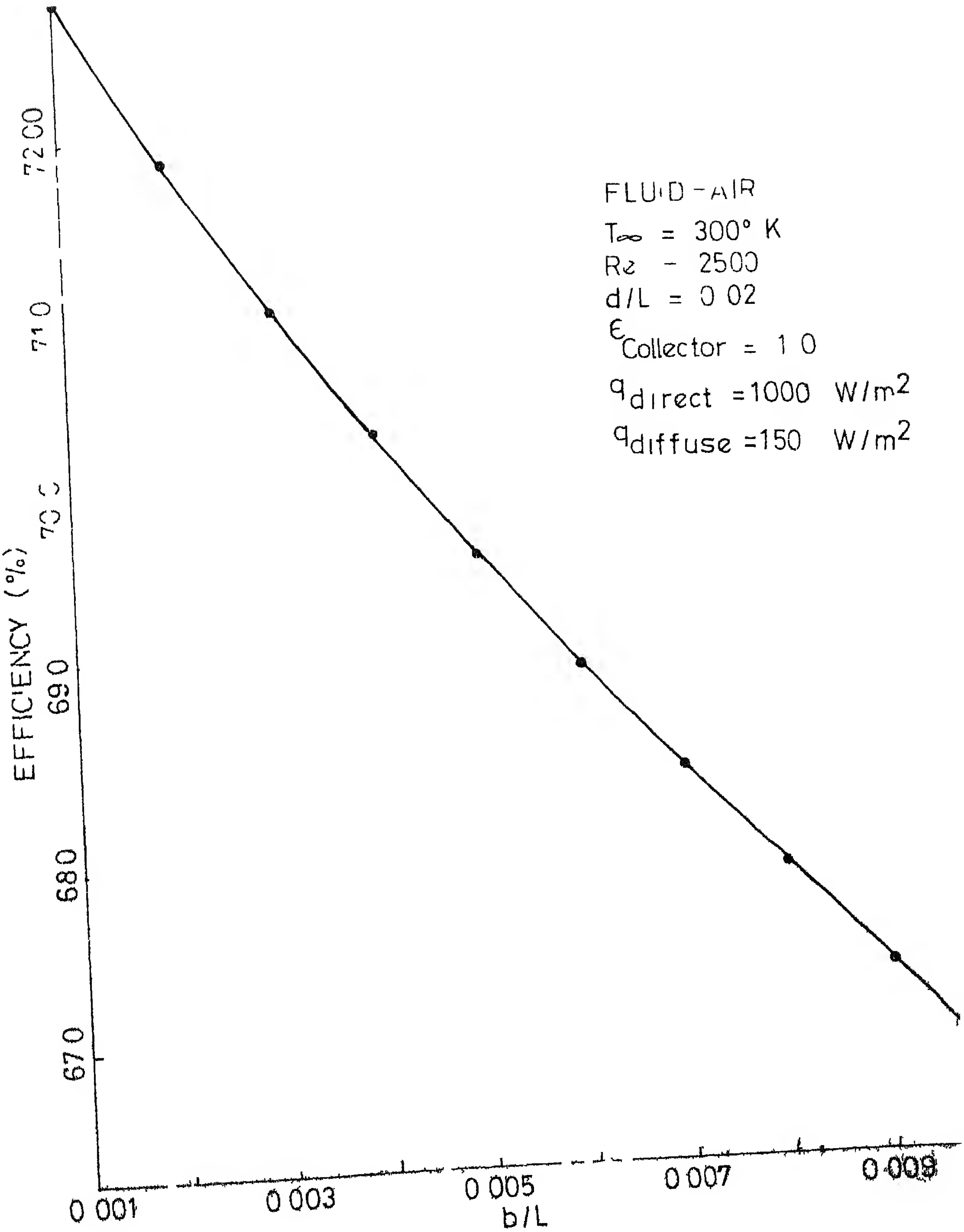

FIG 4 11 EFFECT OF GLASS PLATE THICKNESS

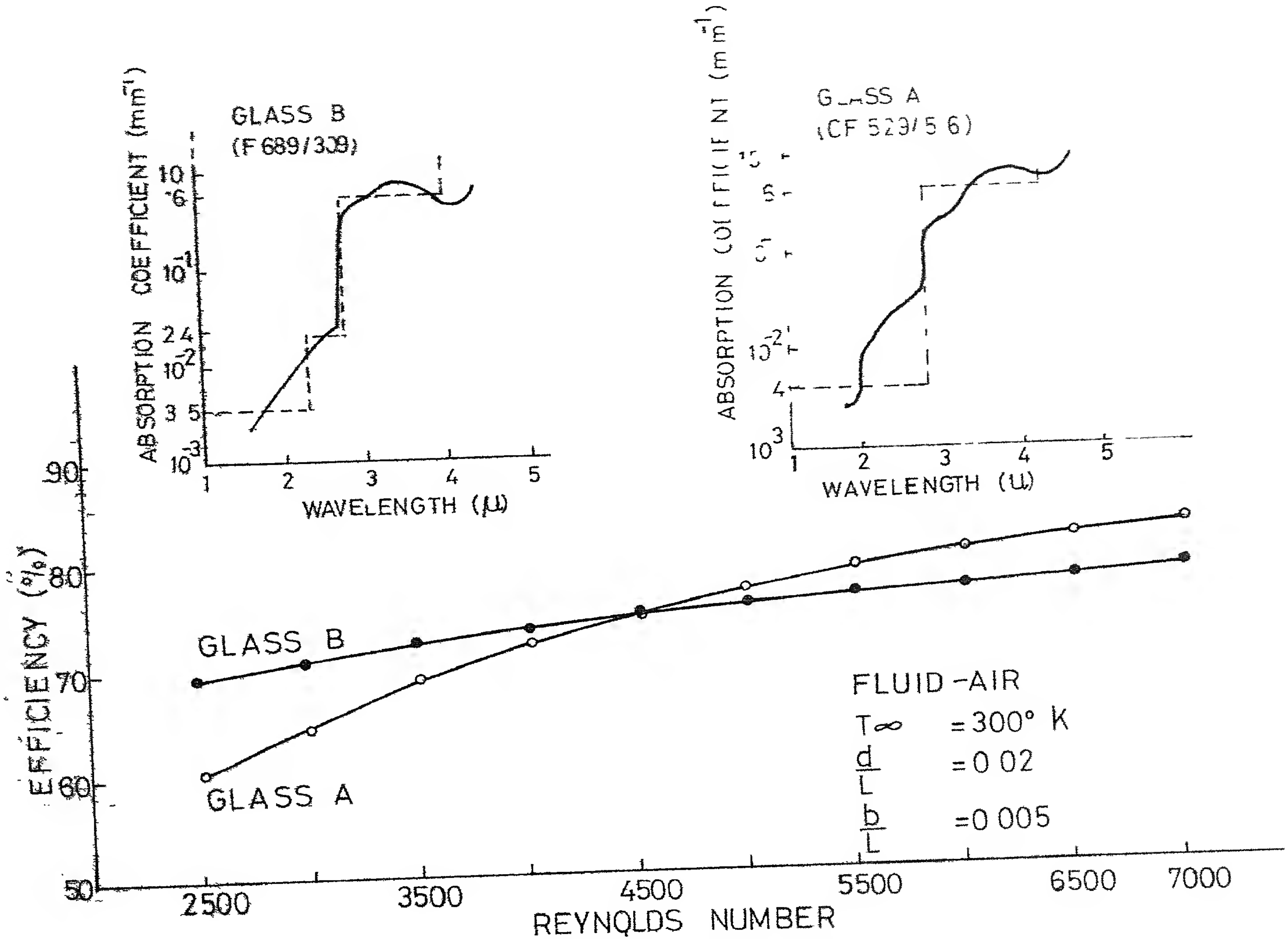

FIG 4·12 FLOW RATE EFFECT FOR TWO TYPES OF GLASS

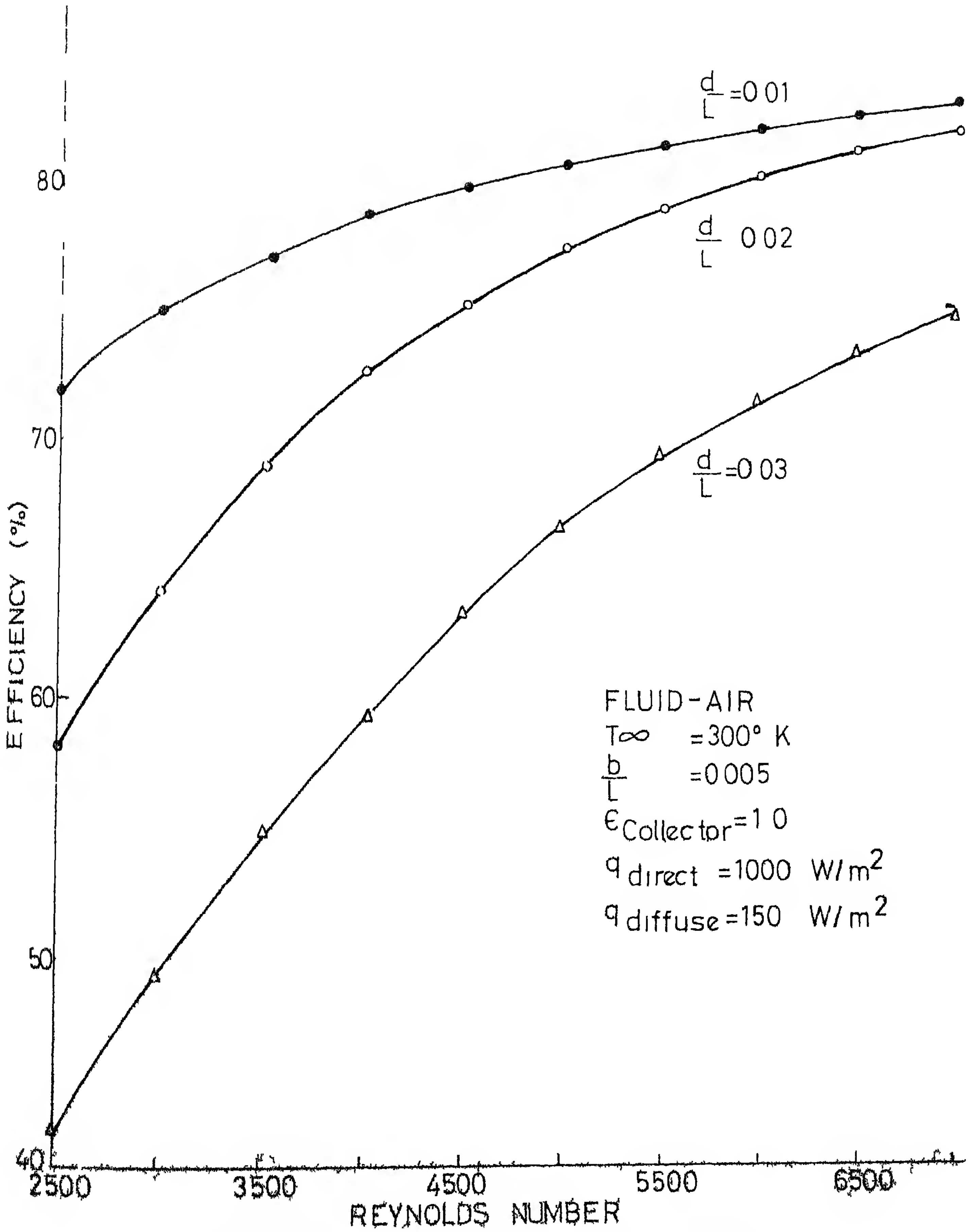

FIG 4 13 EFFECT OF DEPTH OF FLOW

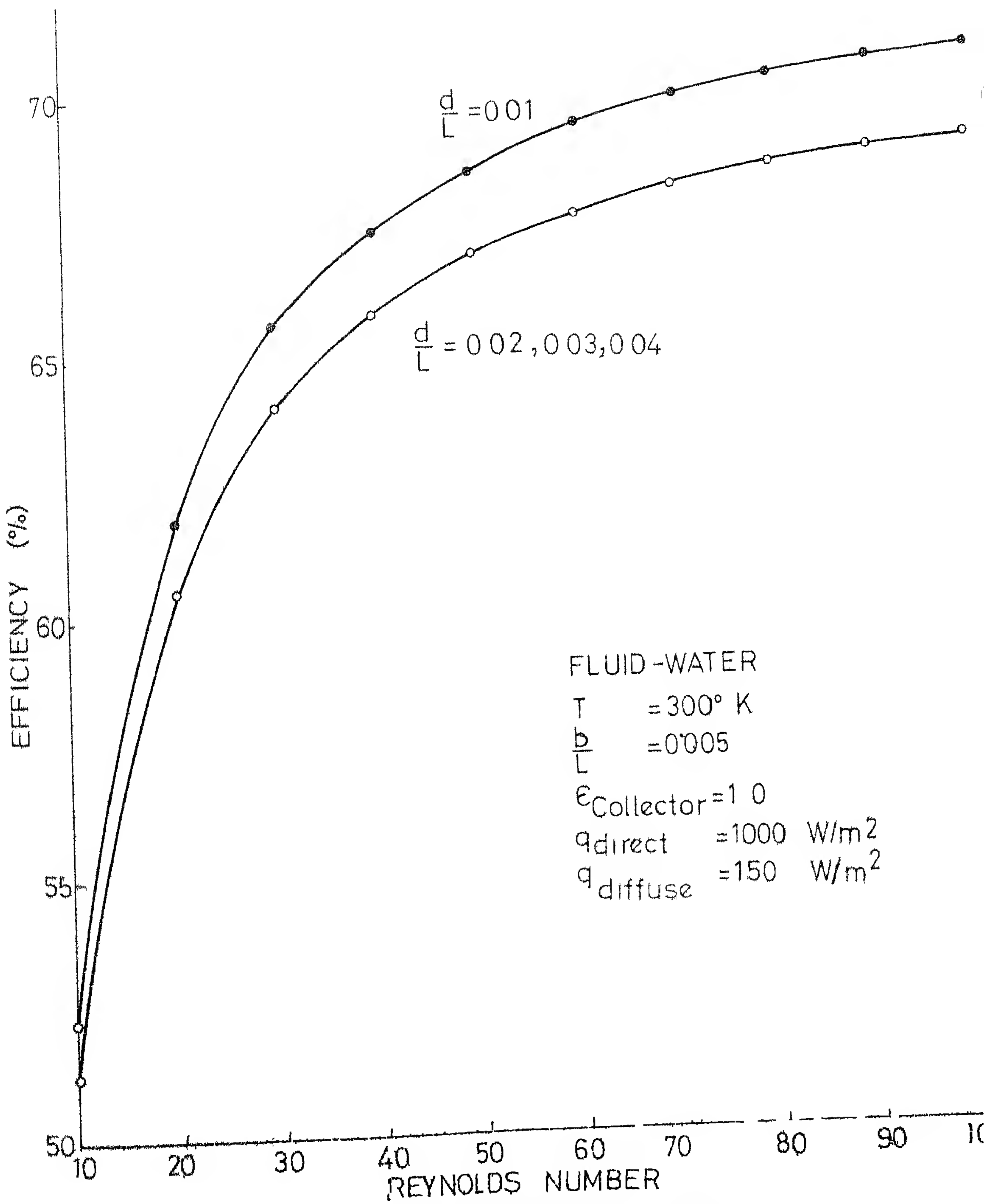

FIG 4 14 EFFECT OF DEPTH OF FLOW

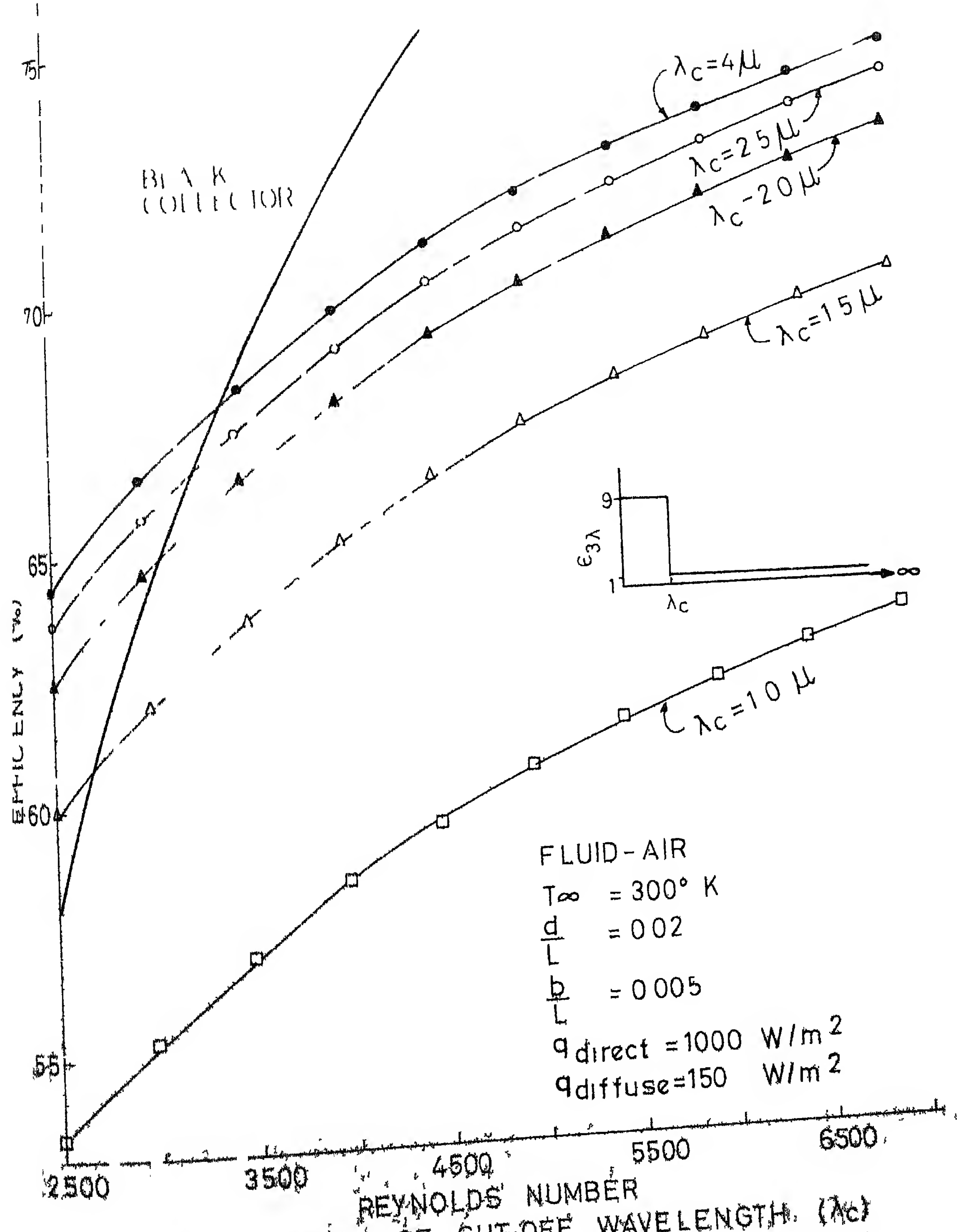

FIG 4:15 EFFECT OF CUT-OFF WAVELENGTH (λc)

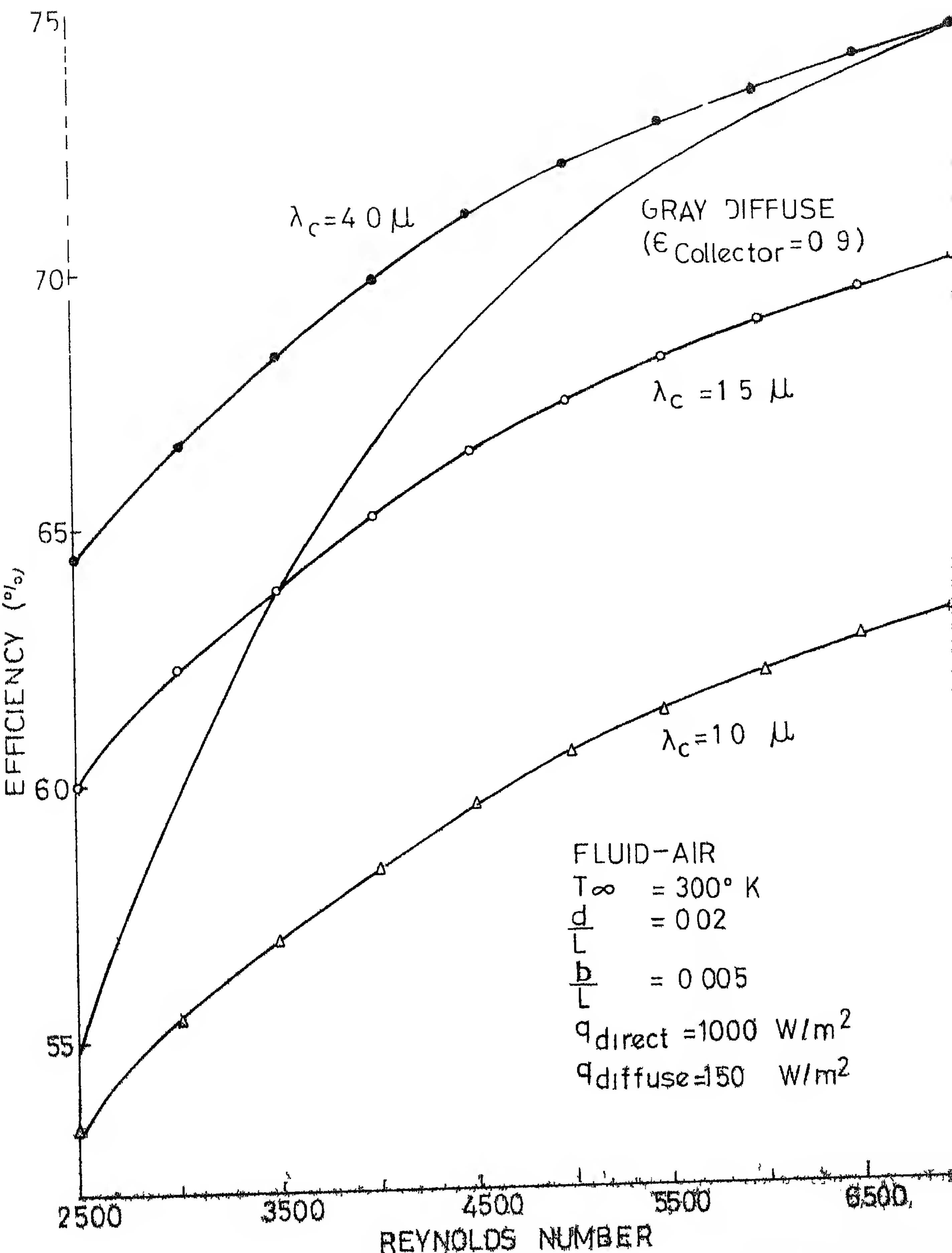

FIG 4.16 COMPARISON OF SELECTIVE SURFACE WITH GRAY DIFFUSE SURFACE

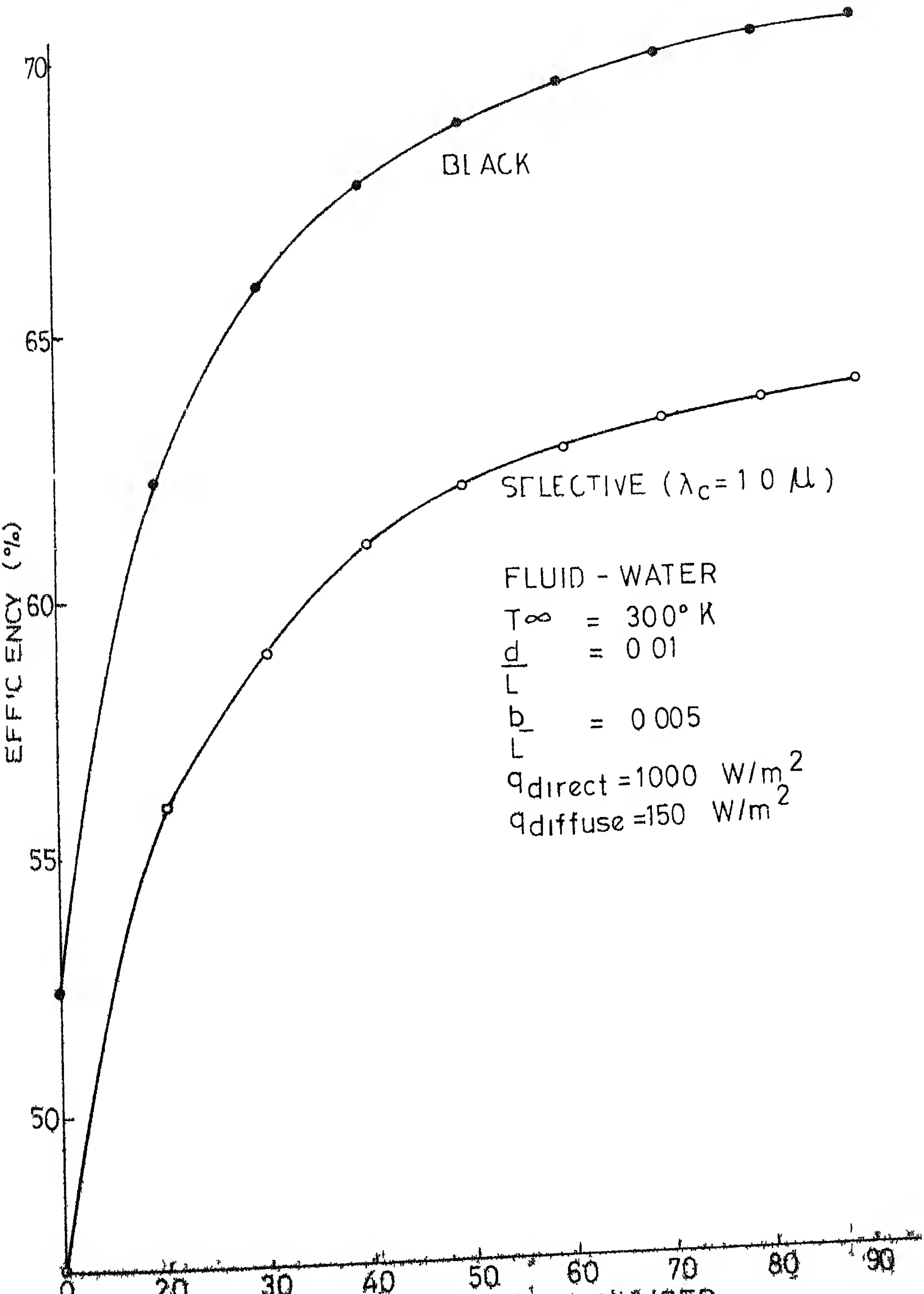

FIG 4'17 COMPARISON OF BLACK AND SELECTIVE COLLECTOR

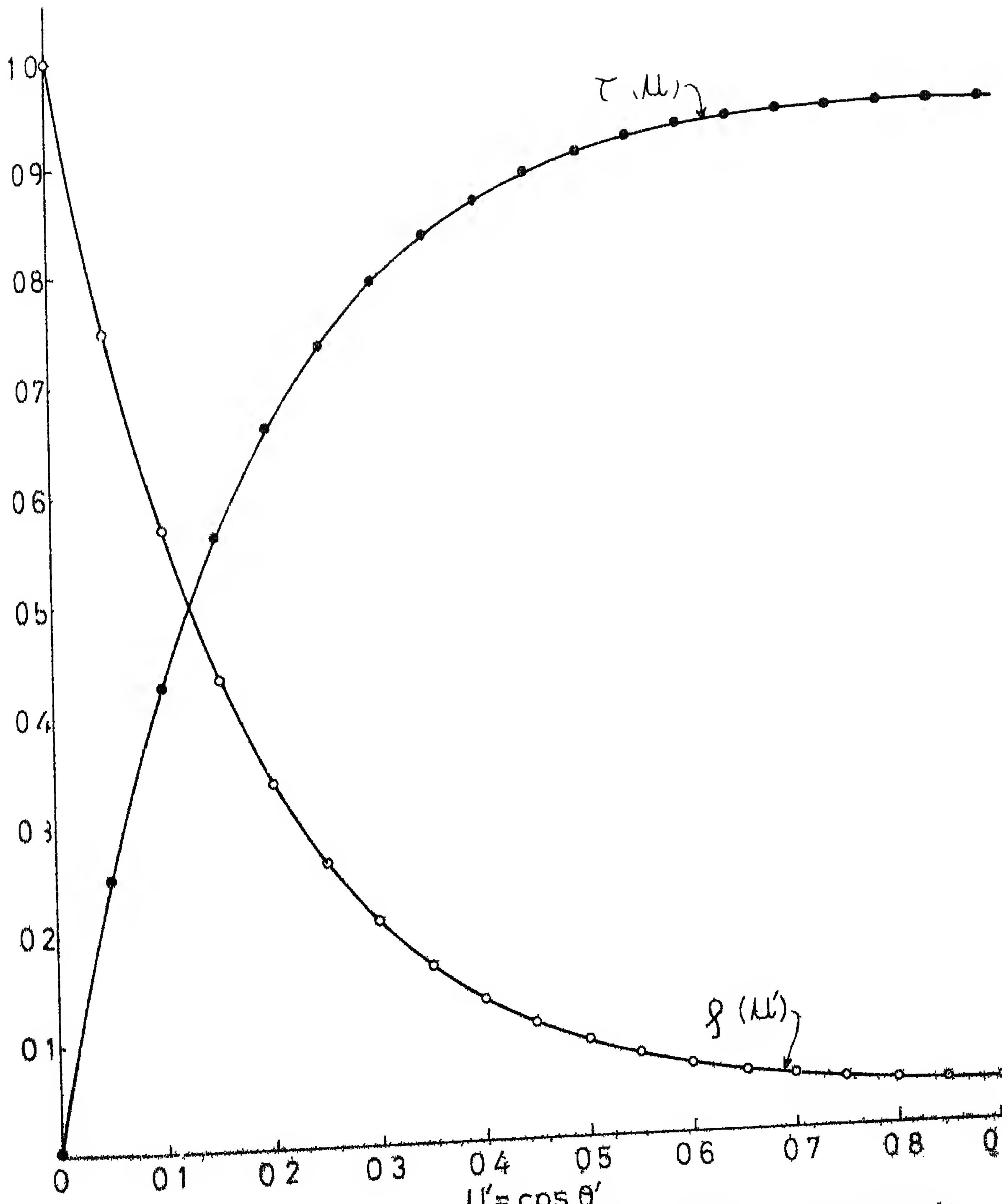

FIG 4 18 TRANSMITTIVITY AND REFLECTIVITY vs ANGLE OF INCIDENCE

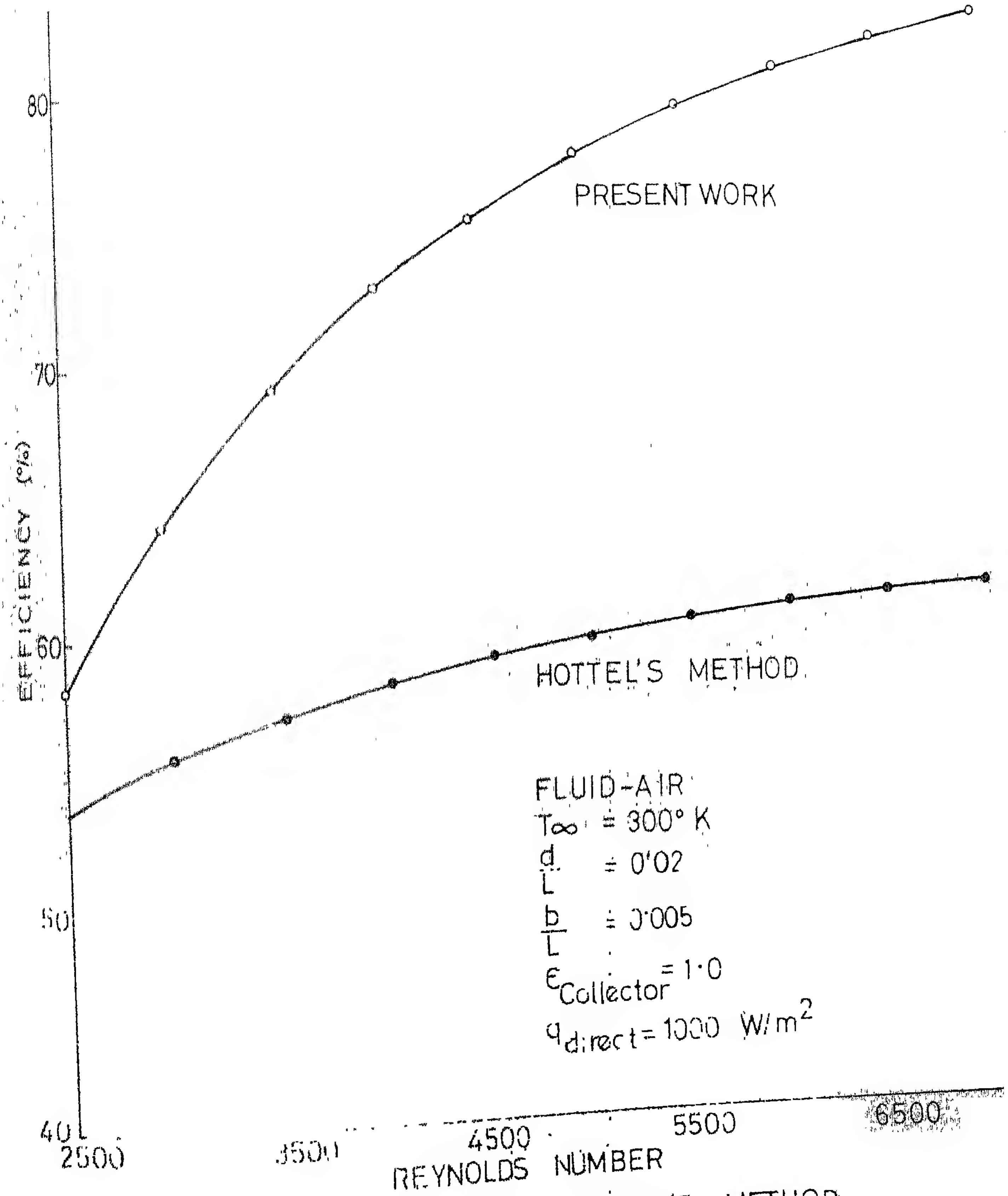

FIG 4.19 COMPARISON WITH HOTTEL'S METHOD